Dynamic Evaluation of Corporate Distress Prediction Models

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Abstract: The design of reliable models to predict corporate distress is crucial as the likelihood of filing for bankruptcy increases with the level and persistency of distress. Although a large number of modelling and prediction frameworks for corporate failure and distress has been proposed, the relative performance evaluation of competing prediction models remains an exercise that is mono-criterion in nature, which leads to conflicting rankings of models. This methodological issue is addressed by Mousavi et al (2015) by proposing an orientation-free super-efficiency data envelopment analysis model as a multi-criteria assessment framework. This data envelopment analysis (DEA) model is static in nature. In this research, we propose a dynamic DEA framework to assess the relative performance of an exhaustive range of distress prediction models and rank them accordingly. In addition, we address several research questions including how robust is the out-of-sample performance of dynamic distress prediction models relative to static ones with respect to sample type and sample period length? and to what extend the choice of distress definition affects the ranking of competing prediction models before, during, and after an important event?

Keywords: Corporate Failure Prediction; Bankruptcy; Performance Criteria; Performance Measures; Data Envelopment Analysis; Dynamic DEA, Slacks-Based Measure

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1. Introduction

Predicting bankruptcy or corporate failure before it happens has such economic benefits for a range of stakeholders (e.g., managers, investors, auditors, regulators) that a large number of prediction models have been designed. In practice, managers could use distress prediction models as early warning systems to take proper preventive actions against bankruptcy. From a conceptual point of view, failure and distress predictions are classification problems, which use a number of features – often extracted from accounting, market, or macroeconomic information – to classify firms into one out of two or more risk categories. During the last decades, numerous studies have employed different types of prediction models or methods from fields such as probability and statistics, operations research, and artificial intelligence – for a detailed classification of distress prediction models, we refer the reader to Aziz and Dar (2006), Bellovary et al. (2007) and Abdou and Pointon (2011).

With the increasing number of prediction models, a strand of the literature has focused on assessing the performance of these models and identifying the factors that drive performance such as modelling frameworks, features selection, estimation methods, sampling, and performance criteria and their measures (Zhou, 2013; Mousavi et al., 2015). As demonstrated by Mousavi et al. (2015), the performance of prediction models is not only dependent on the nature of the modelling frameworks and the type of features, but also on the performance evaluation process and the underlying performance evaluation methodology (e.g., mono-criterion methodologies, multi-criteria methodologies) and the performance criteria and measures with which it is fed. In fact, recent comparative studies have compared the performance of competing failure prediction models grounded into different modelling frameworks (e.g., Wu et al., 2010; Fedorova et al., 2013; Bauer and Agarwal, 2014; Mousavi et al., 2015) and using alternative sampling techniques (e.g., Gilbert et al., 1990; Neves and Vieira, 2006; Zhou, 2013), various features (e.g., Tinoco and Wilson, 2013; Trujillo-Ponce et al., 2014; Mousavi et al., 2015), different feature selection procedures (e.g., Tsai, 2009; Unler and Murat, 2010) and a range of performance criteria (e.g., discriminatory power, calibration accuracy, information content, correctness of categorical prediction) and their measures along with different performance evaluation methodologies (Mousavi et al., 2015).

Our survey of the literature on comparative studies of failure prediction models revealed a variety of shortcomings that prevent practitioners from an efficient ranking of models. As pointed out by Bauer and Agarwal (2014), the literature on comparative studies suffers from two main drawbacks. First, most of the existing studies failed to have a comprehensive comparison between all types of prediction models; i.e., traditional statistical models, contingent claims analysis (CCA) models, and survival analysis (SA) models. Second, the existing literature has used a restricted number of criteria to evaluate the performance of competing models. To have a more comprehensive comparative assessment, Bauer and Agarwal (2014) evaluated the performance of Taffler (1983), Bharath and Shumway (2008) and Shumway (2001) as representative of the traditional statistical models, CCA models and SA models, respectively. Further, they applied three types of criteria; namely, discriminatory power, information content, and correctness of categorical prediction to compare the performance of these models. On the other hand, Mousavi et al. (2015) emphasized a methodological shortcoming in comparative studies arguing that although some studies consider multiple criteria and related measures to compare competing models, the nature of the comparison exercise remains mono-criterion, as they use a single measure of a single criterion at a time. The drawback of this mono-criterion approach is that the rankings corresponding to different criteria are often different (e.g., Bandyopadhyay, 2006; Theodossiou, 1991; Tinoco and Wilson, 2013), which result in a situation where one cannot make an informed decision as to which model performs best when taken all criteria into consideration. To overcome this methodological drawback, Mousavi et al. (2015) proposed a multi-criteria assessment framework; namely, an orientation-free super-efficiency data envelopment analysis. Finally, Zavgren (1983) argued that most traditional failure and distress prediction models are based on the assumption that the relationship between the dependent variable (e.g., probability of failure) and all independent variables (e.g., accounting and market information) is stable over time. Empirical studies, however, indicate that this stability is highly arguable (e.g., Charitou et al., 2004; du Jardin and Séverin, 2012) and that the performance of models is sensitive to changes in macroeconomic conditions (Mensah, 1984; Platt et al., 1994). For example, the logit model of Ohlson (1980) performs better in the mid- to late 1980s, whereas the SA model of Shumway (2001) outperforms other models in the 2000s. The changes in patterns of accounting- and market-based information during time suggest that prediction models need to be re-estimated frequently to encompass the most recent patterns of information (Grice and Ingram, 2001). In this research, we argue that another shortcoming of the existing literature lies in the use of static performance evaluation frameworks to compare prediction models, and we propose a dynamic multi-criteria performance evaluation framework.

Recent studies have substituted financial distress for corporate failure in the implementation of failure prediction models (e.g., Tinoco and Wilson, 2013; Geng et al., 2015; Wanke et al., 2015; Laitinen and Suvas, 2016). Financial distress refers to the inability of a company to pay its financial obligations as they mature (Beaver, 1966). Obviously, the financial situation of a distressed company differs from a healthy one suggesting that, while a company moves toward deterioration, its financial features shift towards the characteristics of failed firms. This movement towards failure is a process that could take several time periods (e.g., years) and manifest itself through a variety of signals, which could prevent failure, if predicted with a reasonable level of accuracy. In this research, in addition to proposing new models to predict distress or detect its signals, we propose a dynamic multi-criteria framework for assessing and monitoring the performance of distress prediction models, which, as a by-product, allows one to detect signals of distress. To the best of our knowledge, no previous research proposed a dynamic framework for the performance evaluation and monitoring of prediction models. In practice, such a framework for the early detection of signs of distress is both necessary and beneficial.

In this paper, we contribute to the academic literature in several respects. First, following the lead of Xu and Ouenniche (2012) and Mousavi et al. (2015) who proposed static multi-criteria frameworks for assessing the relative performance of prediction models, we propose a new dynamic multi-criteria framework for assessing and monitoring the relative performance of prediction models over time and ranking them. Second, we consider a more in-depth classification of statistical distress prediction models and perform an exhaustive evaluation taking into account the most popular models of each class. In sum, we assess the performance of univariate discriminant analysis (UDA), multivariate discriminant analysis (MDA), linear probability analysis (LPA), probit analysis (PA) and logit analysis (LA) models as traditional techniques;

Black-Scholes-Merton (BSM)-based models, naïve BSM-based models, and naïve down-and-out call (DOC) barrier option models as contingent claims analysis (CCA) models; and duration independent and duration dependent survival analysis (SA) models. To best of our knowledge, this study is the first to propose the Cox model with time-varying variables using UK data for distress prediction, or equivalently estimating distress probabilities. To date, this study provides the most comprehensive empirical comparative analysis of statistical distress prediction models. Third, we provide answers to several important research questions using a rolling horizon sampling framework and a dynamic performance evaluation and monitoring framework: What category of information or combination of categories of information enhances the predictive ability of models best? and How the out-of-sample performance of dynamic distress prediction models compare to the out-of-sample performance of static ones with respect to sample type and sample period length?

The rest of the paper unfolds as follows. Section 2 reviews the literature on advances in and comparative studies on distress prediction models. Section 3 describes the proposed dynamic multi-criteria framework; namely, a non-oriented super-efficiency Malmquist DEA, for the comparison of prediction models. Section 4 provides details on our experimental design including data, sample selection, and the variety of distress prediction models compared as part of this study. Section 5 summarises our empirical results and discusses our findings. Finally, section 6 concludes the paper.

2. Comparative studies on distress prediction models

In this section, we provide a concise account of advances on distress prediction modeling (see section 2.1) along with a detailed survey of comparative studies (see section 2.2).

2.1. Advances in distress prediction models

Failure and distress prediction models could be divided into several categories depending on the choice of the classification criteria. In this paper, we focus on a variety of models but the artificial intelligence and mathematical programming ones. In sum, we consider the first generation of models; namely, discriminant analysis (DA) models (e.g., Beaver, 1966, 1968; Altman, 1968; Deakin, 1972; Blum, 1974; Altman et al., 1977), the second generation of models; namely, probability models such as linear probability (LP) models (e.g., Meyer and Pifer, 1970), logit analysis (LA) models (e.g., Martin, 1977; Ohlson, 1980), and probit analysis (PA) models (e.g., Zmijewski, 1984), and the third generation of models; namely, survival analysis (SA) models (e.g., Lane et al., 1986; Crapp and Stevenson, 1987; Luoma and Laitinen, 1991; Shumway, 2001) and contingent claims analysis (CCA) models (e.g., Hillegeist et al., 2004; Bharath and Shumway, 2008).

Beaver (1966, 1968) is the pioneering study which proposed a univariate discriminant analysis model fed with financial ratios information to predict failure. However, the first multivariate study was undertaken by Altman (1968) who estimated a score, commonly referred to as a "Z-score", as a proxy of the financial situation of a company using multivariate discriminant analysis (MDA). The suggested MDA technique was frequently used in later studies (e.g., Deakin, 1972; Blum, 1974; Altman et al., 1977; Altman, 1983). The majority of subsequent studies applied the second generation models; that is, linear probability models (e.g., Meyer and Pifer, 1970), logit models (e.g., Martin, 1977; Ohlson, 1980), and probit models (e.g., Zmijewski, 1984). These first and second generations of models could be viewed as empirical models in that they are driven by practical considerations such as an accurate prediction of the risk class or an accurate estimate of the probability of belonging to a risk class; in sum, the choice of the explanatory variables is driven by the predictive performance of the models. These models and their usage in some previous studies are not without limitations. In fact, some of the assumptions underlying the modelling frameworks may not be reasonably satisfied for some datasets, on one hand, and earliest studies restricted the type of information to accounting-based one. In addition, these models are static in nature and therefore fail to properly account of changes over time in the profiles of companies. The third generation of models; namely, survival analysis (SA) models and contingent claims analysis (CCA) models overcome some of these issues. In fact, the underlying modelling frameworks of both SA models and CCA models are dynamic by design. In addition, most previous studies made use of additional sources of information to enhance the performance of these models; namely, market-based information (e.g., Hillegeist et al., 2004; Bharath and

Shumway, 2008) and macroeconomic information (e.g., Tinoco and Wilson, 2013; Kim and Partington, 2014; Charalambakis and Garrett, 2015) although one might argue that the approximation process of unobservable variables (e.g., volatility, expected return, market value of assets) is not free of potential measurement errors (Aktug, 2014). To be more specific, SA models are used to estimate time-varying probabilities of failure. Despite the application of SA models in failure prediction dates back to the mid-1980s (e.g., Lane et al., 1986; Crapp and Stevenson, 1987; Luoma and Laitinen, 1991), Shumway (2001) was the pioneering study which made its use popular by providing an attractive estimation methodology based on an equivalence between multi-period logit models and a discrete-time hazard model. Thereafter, the suggested discrete-time hazard model – also referred to as a discrete-time logit model – was frequently used in later studies (e.g., Chava and Jarrow, 2004; Wu et al., 2010; Tinoco and Wilson, 2013; Bauer and Agarwal, 2014) to estimate the coefficients of time-varying accounting and market-based covariates of SA models. Unlike, the first generation models, the second generation models, and SA models, which are empirical models, CCA models – also referred to as Black-Scholes-Merton (BSM)-based models – are theoretically grounded. In fact, these models are grounded into option-pricing theory, as set out in Black and Scholes (1973) and Merton (1974) whereby the equity holders' position in a firm is assumed to be the long position in a call option. Therefore, as suggested by McDonald (2002), the probability of failure could be interpreted as the likelihood that the value of firm's assets will be less than the face value of firm's liabilities at maturity; i.e., the call option expires worthless. These models make use of market-based information by incorporating company stock returns and their volatility in estimating the probability of failure (Hillegeist et al., 2004; Bharath and Shumway, 2008). Like any modelling framework, CCA models are not without their limitations. For example, CCA models implicitly assume that the liabilities of the firm have the same maturities, which in practice is a limitation (Saunders and Allen, 2002).

2.2. Comparative studies of failure prediction models

This section provides a survey on the studies, which focus on the comparison of different types of failure or distress prediction models; namely, the first generation of models, the second generation of models, and the third generation of models. Our survey focus is on models and performance criteria and their measures, which have been applied by the existing literature on the evaluation of competing prediction models.

Comparison between first and second generation models: Before the breakthrough model of Shumway (2001), the first and second generations of models were the prevailing techniques in classification. Since the implementation of DA in failure prediction by Beaver (1966) and Altman (1968) to the early 1980s, MDA was the superior method for predicting corporate failure. In fact, ease of use and interpretation were the main reasons of the popularity of DA. However, the validity of these models depends on the extent to which the underlying assumptions (i.e., multivariate normality, equal groups' variance-covariate matrices) hold in a dataset. From the 1980s to 2001, LA models (introduced by Ohlson, 1980) and PA models (introduced by Zmijewski, 1984) became the prevailing techniques. Despite the fact that probability models are more attractive from a practical perspective in that the underlying assumptions are less restrictive, most comparative studies have indicated that the prediction powers of LA models and PA models are similar to those of DA models (e.g., Press and Wilson, 1978; Collins and Green, 1982; Lo, 1986). A notable exception is Lennox (1999) who suggested that well-specified probit and logit models outperform DA models.

Comparison between first and second-generation models and survival analysis models: From a conceptual perspective, SA models are superior to discriminant analysis models and probability models, because of their dynamic nature. However, empirical results across several comparative analyses seem to report mixed findings. From an empirical perspective, the features of a modelling framework design that are not being fully supported or exploited by the dataset under consideration nullify its conceptual advantage. In sum, the choice combination of a modelling framework and the features to feed into it has a more significant role in enhancing or downgrading prediction performance.

For example, Luoma and Laitinen (1991) compared the performance of a semiparametric Cox hazard model with a DA model and an LA model – all models fed with accounting based information – with respect to type I and type II errors as measures of correctness of categorical prediction. The results suggested that the developed SA was inferior to both DA and LA models with respect to type I and type II errors. Further, their research was limited with respect to the number of criteria, since they only used correctness of categorical prediction.

Shumway (2001) proposed a discrete-time SA model – using a multi-period logit estimation technique – for failure prediction and compared its performance with the performance of DA, LA and PA using overall correct classification rate (OCC) as a measure of correctness of categorical prediction. The results indicate that an SA model, which encompasses both accounting and market information (respectively, only accounting information) outperforms (respectively, underperforms) DA, LA and PA models. However, with respect to the choice of performance criteria and their measures, this study is also restricted to correctness of categorical prediction as a criterion and overall accuracy – also known as overall correct classification rate – as its measure.

Comparison between first and second generation models and contingent claims models; Hilligeist et al. (2004) compared the performance of an BSM-based model with two types of representative models of the first and second generation of models; namely, MDA and LA models (Altman, 1968; Ohlson, 1980), respectively. They used Log-Likelihood and Pseudo-R^2 as measures of information content to evaluate the performance of these models. The results suggested that the BSM-based model outperforms both the original and the refitted versions of Altman (1968) and Ohlson (1980) models on information content. Furthermore, they found out that the original Altman (1968) with coefficients estimated with a small data set from decades earlier outperformed the refitted one with updated coefficients using recent data suggesting that refitting models with more recent data does not necessarily improve performance. However, this study is restricted to one criterion; i.e., information content, for comparing the performance of models.

Reisz and Perlich (2007) compared the performance of three contingent claims models; namely, a BSM model, a KMV model developed by KMV Corporation in 1993 and then acquired by Moody's Corporation in 2002, and a Down-and-Out Call option (DOC) model, with the MDA model of Altman (1968). Recall that the KMV model – also referred to as the Expected Default Frequency (EDF) model – is actually a four-step procedure based on Merton’s framework, which determines a default point, estimates asset value and volatility, computes distance to default (DD), and converts DD into expected default frequency (EDF). They use ROC as a measure of discriminatory power and Log-Likelihood as a measure of information content. They found out that the DOC model outperforms the other types of contingent claims models as well as MDA for 3-, 5- and 10-year ahead failure prediction. Unexpectedly, Altman (1968) outperforms all contingent claims models for 1-year ahead failure prediction. Although this study encompassed two types of criteria; i.e., discriminatory power and information content, the comparison is somehow incomplete as the log-likelihood cannot be computed for Altman's model.

Agarwal and Taffler (2008) compared the performance of two types of BSM models; namely, Hillegeist et al. (2004) and Bharath and Shumway (2008), with the MDA model of Taffler (1983) with respect to ROC as a measure of discriminatory power, Log-likelihood and Pseudo-R2 as measures of information content, and return on assets (ROA) and return on risk weighted assets (RORWA) as measures of economic value, given different costs of misclassification. The empirical results showed that the MDA model outperforms Hillegeist et al. (2004) significantly on ROC as a measure of discriminatory power. Meanwhile, MDA model does not outperform Bharath and Shumway (2008) significantly on ROC. On the other hand, with respect to Log-likelihood as a measure of information content, Hillegeist et al. (2004) performs significantly better than Bharath and Shumway (2008) and MDA model, respectively. However, Pseudo-R^2 was higher for Taffler (1983) compared to BSM models, which suggests that these two information content measures carry different elements of information. Furthermore, taking into account differences in misclassification costs, they compared the economic benefit of applying Bharath and Shumway (2008) or Taffler (1983) as classifiers using the approach proposed by Blochlinger and Leippold (2006). The results suggest that the MDA model of Taffler (1983) outperforms BSM-based models. It is worth to mention that with respect to the number of criteria, this study was innovative in its era since three criteria; namely, the correctness of categorical prediction, discriminatory power and information content, were used for evaluating models.

Comparison between contingent claims models and survival analysis models: Campbell et al.(2008) proposed a duration-dependent SA model and evaluated its performance with the performances of a KMV model and the duration-independent SA model of Shumway (2001) using log-likelihood and pseudo-R^2 as measures of information content. The results indicate that their

SA model outperforms both the SA model of Shumway (2001) and the KMV model. However, this study fails to incorporate more criteria for comparing the performance of models.

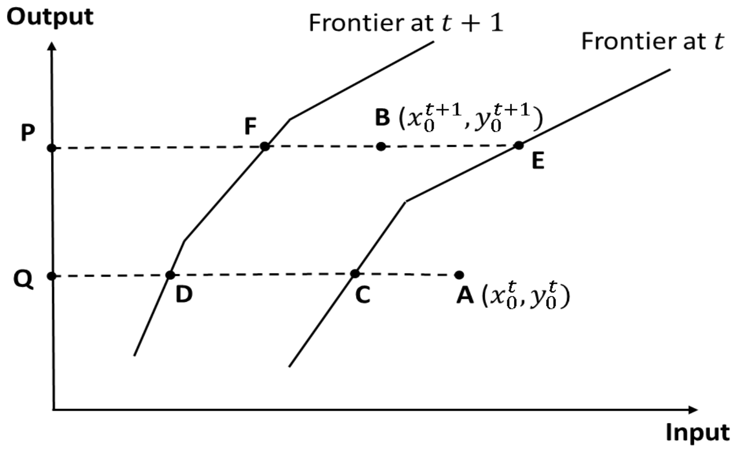
Comparison between first, second and third generations of models: Wu et al. (2010) compared the performance of the MDA model of Altman (1968), the LA model of Ohlson (1980), the PA model of Zmijewski (1984), the duration-independent SA model of Shumway (2001) and the BSM model of Hillegeist et al. (2004). The results indicate that, with respect to Log-likelihood and Pseudo-R2 as measures of information content, the discrete time SA model of Shumway outperforms LA, PA, BSM and MDA models, respectively. Unexpectedly, with respect to overall correct classification rate as a measure of correctness of categorical prediction, Ohlson model of LA outperforms MDA, PA, BSM, SA models, respectively, under a rolling window implementation. For ROC as a measure of discriminatory power, the authors failed to take account of BSM model, the result suggest that the duration-independent SA model of Shumway performs better than LA, MDA, and PA, respectively. Referring to the number of criteria, this study puts comparison into effect with three types of criteria, namely, the correctness of categorical prediction, discriminatory power, and information content.

To conclude this section, we would like to refer the reader to Appendix A for a summary table of the literature on comparative analyses of failure models. We also refer the reader to Appendix C of Mousavi et al (2015) for a sample of typical performance criteria and their measures used in assessing failure prediction models.

3. A Dynamic Framework for Assessing Distress Prediction Models: Non-Oriented Super-Efficiency Malmquist DEA

Malmquist productivity index is a multi-criteria assessment framework for performing performance comparisons of DMUs over time. Fare et al. (1992, 1994) employed DEA to extend the original Malmquist (1953) and construct the DEA-based Malmquist productivity index as the product of two components, one measuring the efficiency change (EC) of DMU with respect to the efficiency possibilities defined by the frontier in each period (also referred to as caching-up to the frontier), and the other measuring the efficient frontier-shift (EFS) between the two time periods t and $t+1$ (also referred to as change in the technical efficiency evaluation).

Figure 1: Efficiency Change and Efficient Frontier-Shift



Let x_{i0}^t denote the ith input and y_{r0}^t denote the rth output for DMU_0, both at period t. The Figure 1 shows the change of efficiency of DUM_0 from point A (with respect to efficient frontier at period t) to point B (with respect to efficient frontier at period $t+1$) assuming to have one input and one output. The efficiency change (EC) component is measured by the following formula:

$$\text{EC} = \frac{{}^{PF}/_{PB}}{{}^{QC}/_{QA}} = \frac{\text{Efficiency of DMU}_0 \text{ with respect to the period } t+1}{\text{Efficiency of DMU}_0 \text{ with respect to the period } t} \tag{1}$$

Let $\Delta^{t_2}((x_0, y_0)^{t_1})$ denote the efficiency score of DUM with x_0 input and y_0 output at period t_1 (say, $DMU(x_0, y_0)^{t_1}$) relative to frontier t_2. Replacing t_1 and t_2 with t and $t+1$, respectively, the EC effect (say, α) can be presented as:

$$EC: \quad \alpha = \frac{\Delta^{t+1}((x_0, y_0)^{t+1})}{\Delta^t((x_0, y_0)^t)} \tag{2}$$

Thus, $EC > 1$ shows an improvement in relative efficiency from period t to $t+1$, while $EC = 1$ and $EC < 1$ shows stability and deterioration in relative efficiency, respectively.

Also, Figure 1 indicates that the reference point of (x_0^t, y_0^t) moved from C on the frontier of period t to D on the frontier of period $t+1$. Therefore, the efficient frontier-shift (EFS) effect at (x_0^t, y_0^t) is equivalent to:

$$EFS_t = \frac{QC}{QD} = \frac{QC/_{QA}}{QD/_{QA}} \tag{3}$$

$$= \frac{\text{Efficiency of } (\mathrm{x}_0^{\mathrm{t}}, \mathrm{y}_0^{\mathrm{t}}) \text{ with respect of the period t frontier}}{\text{Efficiency of } (\mathrm{x}_0^{\mathrm{t}}, \mathrm{y}_0^{\mathrm{t}}) \text{ with respect of the period } \mathrm{t}+1 \text{ frontier}}$$

Similarly, the EFS effect at (x_0^{t+1}, y_0^{t+1}) is equivalent to:

$$EFS_{t+1} = \frac{BF}{BD} = \frac{BF/_{BQ}}{BD/_{BQ}} \tag{4}$$

$$= \frac{\text{Efficiency of } (\mathrm{x}_0^{\mathrm{t+1}}, \mathrm{y}_0^{\mathrm{t+1}})\text{with respect of the period t frontier}}{\text{Efficiency of } (\mathrm{x}_0^{\mathrm{t+1}}, \mathrm{y}_0^{\mathrm{t+1}})\text{with respect of the period } \mathrm{t}+1 \text{ frontier}}$$

The EFS component is measured by the geometric mean of EFS effect at (x_0^t, y_0^t) (say, EFS_t) and EFS effect at (x_0^{t+1}, y_0^{t+1}) (say, EFS_{t+1});

$$EFS = [EFS_t \times EFS_{t+1}]^{1/2} \tag{5}$$

Using our notation, the EFS effect can be expressed as:

$$EFS: \quad \beta = \left[\frac{\Delta^t((x_0, y_0)^t)}{\Delta^{t+1}((x_0, y_0)^t)} \times \frac{\Delta^t((x_0, y_0)^{t+1})}{\Delta^{t+1}((x_0, y_0)^{t+1})}\right]^{1/2} \tag{6}$$

Therefore, the Malmquist Productivity index (MPI) can be written as;

$$MPI = EC \times EFS \tag{7}$$

Using our notation, the MPI can be presented as:

$$MPI: \quad \gamma = \alpha \times \beta \tag{8}$$
$$= \frac{\Delta^{t+1}((x_0, y)^{t+1})}{\Delta^{t}((x_0, y_0)^{t})} \times \left[\frac{\Delta^{t}((x_0, y_0)^{t})}{\Delta^{t+1}((x_0, y_0)^{t})} \times \frac{\Delta^{t}((x_0, y_0)^{t+1})}{\Delta^{t+1}((x_0, y_0)^{t+1})}\right]^{1/2}$$

MPI could be rearranged as;

$$\gamma = \left[\frac{\Delta^{t}((x_0, y_0)^{t+1})}{\Delta^{t}((x_0, y_0)^{t})} \times \frac{\Delta^{t+1}((x_0, y_0)^{t+1})}{\Delta^{t+1}((x_0, y_0)^{t})}\right]^{1/2} \tag{9}$$

This explanation of MPI could be interpreted as the geometric mean of efficiency change measured by period t and $t+1$ technology, respectively. $MPI > 1$ shows an improvement in the total factor productivity of DMU_0 from period t to $t+1$, while $MPI = 1$ and $MPI < 1$ shows stability and deterioration in total factor productivity, respectively.

Comment 1: Caves et al. (1982) introduced a distance function, $\Delta(.)$, to measure technical efficiency in the basic CCR model (Charnes et al., 1978). Though, in the non-parametric framework, instead of using a distance function, DEA models are implemented. For example, Fare et al. (1994) used input (or output) oriented radial DEA to measure the MPI. However, the radial model faces a lack of attention to slacks, which could be overcome using Slack-based non-radial oriented (or non-oriented) DEA model (Tone, 2001, 2002).

In this study, we use the non-radial (slack-based measure), non-oriented super- efficiency DEA (Tone, 2002, 2001) Malmquist index to evaluate the performance of competing distress prediction models. The reason to choose an orientation-free evaluation is that we aim to evaluate distress prediction models, and thus, the choice between input-oriented or output-oriented analysis is irrelevant. Further, our study is under variable return to scale (VRS) assumption, where input-oriented and output-oriented analysis may result in different scores and rankings of DMUs. On the other hand, the reason to choose non-radial framework is that, radial DEA models may be infeasible for some DMUs; therefore, ties would stay in rankings. Moreover, radial DEA models overlook possible slacks in inputs and outputs, and therefore, would possibly over-estimate the efficiency scores by ignoring mix efficiency.

Further, basic DEA techniques cannot distinguish between efficiency DMUs (here, distress prediction models) because all their scores are equal to 1 (Anderson and Peterson, 1993). Therefore, we choose super-efficiency DEA framework, as we are interested in acquiring a complete ranking of distress prediction models.

Considering the production possibility set P defined by Cooper et al. (2006) as

$$P = \{(x, y) | x \geq X^{t_1}\lambda, y \leq Y^{t_1}\lambda, 1 \leq e\lambda \leq 1, \lambda \geq 0\}, \qquad (10)$$

SBM-DEA (Tone, 2001) measures the efficiency of DMU $(x_0, y_0)^{t_2}$ $(t_2 = 1{,}2)$ with respect to the benchmark set $(X, Y)^{t_1}(t_1 = 1{,}2)$ using the following linear programing (LP):

$$\Delta^{t_1}((x_0, y_0)^{t_2}) = \min_{\lambda, s^-, s^+} \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}^{t_2}}}{1 + \frac{1}{r}\sum_{i=1}^{r} \frac{s_i^+}{y_{io}^{t_2}}} \qquad (11)$$

$$\begin{gathered}\text{subject to} \quad x_o^{t_2} = X^{t_1}\lambda + s^-, \\ y_0^{t_2} = Y^{t_1}\lambda - s^+ , \\ 1 \leq e\lambda \leq 1, \\ \lambda \geq 0, s^- \geq 0, s^+ \geq 0.\end{gathered}$$

where $\Delta^{t_1}((x_0, y_0)^{t_2})$ is the efficiency score of $DMU(x_0, y_0)^{t_1}$ relative to frontier t_2; $X^{t_1} = \left(x_1^{t_1}, \dots, x_n^{t_1}\right) \in \mathbb{R}^n$ and $Y^{t_1} = (y_1^{t_1}, \dots, y_n^{t_1}) \in \mathbb{R}^n$ are matrices of inputs and outputs at the period t_1, respectively; $s^- \geq 0$ and $s^+ \geq 0$ are the vectors of input surpluses and output shortages in $\mathbb{R}^n$, respectively, and are named *slacks; e* is a row vector with all items equal to one, and λ is a nonnegative vector in $\mathbb{R}^n$.

Or equivalently;

$$\Delta^{t_1}((x_0, y_0)^{t_2}) = \min_{\theta, \eta, \lambda} \frac{\frac{1}{m}\sum_{i=1}^{m} \theta_i}{\frac{1}{r}\sum_{i=1}^{r} \eta_i} \qquad (12)$$

$$
\begin{gathered}
\text{subject to} \quad \theta_i x_{io}^{t_2} \geq \sum\nolimits_{j=1}^{n} x_{ij}^{t_1} \lambda_j \;\; (i = 1, \dots, m), \\
\eta_i x_{io}^{t_2} \geq \sum\nolimits_{j=1}^{n} y_{ij}^{t_1} \lambda_j \;\; (i = 1, \dots, r)\,, \\
\theta_i \leq 1 (i = 1, \dots, m), \eta_i \geq 1 (i = 1, \dots, r), \\
1 \leq e\lambda \leq 1, \\
\lambda \geq 0.
\end{gathered}
$$

where $\theta_i$and η_i are $\left(1 - \frac{s_i^-}{x_{io}^{t_2}}\right)$ and $\left(1 + \frac{s_i^+}{y_{io}^{t_2}}\right)$, respectively.

Referring to equation 9, someone can use equation 11 to estimated $\Delta_0^t(x_0^t, y_0^t)$, $\Delta_0^{t+1}(x_0^{t+1}, y_0^{t+1})$, $\Delta_0^t(x_0^{t+1}, y_0^{t+1})$ and $\Delta_0^{t+1}(x_0^t, y_0^t)$ as four required terms for calculating MPI.

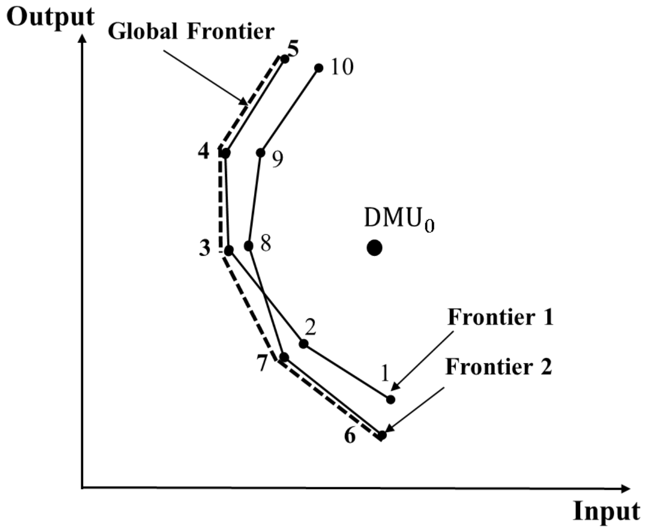
Comment 2: The main objective of this study is to estimate the relative efficiency of $DMUs$ in each period. However, the estimated Malmquist productive index, say, $MPI_0^{t,t+1}$, indicates the change of efficiency score between period t and $t+1$, and should be modified for our purpose. Further, according to Pastor and Lovell (2005), the contemporaneous MPI is not circular, its adjacent period components can give conflicting signals, and it is sensitive to LP infeasibility.

The adjacent reference index, proposed by Fare et al., (1982), suggests multiplying $MPI_0^{t,t+1}$ by $\Delta_0^t(x_0^t, y_0^t)$, which results in the relative efficiency of DMU_0 at period $t+1$ compared to period t. However, the main drawback of this index is that it cannot estimate the relative efficiency score of non-adjacent periods, e.g., period t and $t+2$ or $t+1$ and $t+3$.

To overcome this drawback, Berg et al, (1992) used a fixed reference index, which compares and refers the relative efficiencies of all periods (say, $t\ (t \geq 2)$) to the first period (say, $t = 1$). Therefore, it is possible that the efficiency scores of the periods later than the first one are more than 1 since the technology develops over time. Although, fixed reference index acquire the circularity property with a base period dependence, it remains sensitive to LP infeasibility.

More recently, Pastor and Lovell (2005) suggested a global MPI index, which its components are circular, it provides single measures of productivity change, and it is not susceptible to LP infeasibility. Further, in situation where efficient frontiers of multiple periods cross each other, global index can be measured by the best practices in all periods.

Figure 2: Global Frontier



As Figure 2 presents, the relative efficiency of DMU_0 can be measured in terms of either the frontier of period 1 (consists of four DMUs of 1,2,3,4 and 5) or the frontier of period 2 (consist of four DMUs of 6,7,8,9 and 10). An alternative is the global frontier, which is the combination of the best DMUs in the history, i.e. five DMUs of 6,7,3,4 and 5.

It is argued that if the length of observation period is long enough, the current DMUs would be covered by the best historical DMUs, probably themselves. As a result, the relative efficiency to the global frontier could be considered as an absolute efficiency with the scores less than or equal to 1 (Pastor and Lovell, 2005).

4. Empirical investigation

In this section, we provide the details of our empirical investigation, where we compare the performance of both existing and new distress prediction models using both mono- and multi-criteria performance evaluation frameworks. In the remainder of this section, we shall provide details on our dataset (see section 4.1), features selection (see section 4.2), sampling and fitting choices (see section 4.3), distress prediction models (see section 4.4), and our empirical results and findings (see section 4.5).

4.1. Data

The dataset used in our empirical analysis is chosen as follows. First, we considered all non-financial and non-utility UK companies listed on the London Stock Exchange (LSE) at any time during a 25-year period from 1990 through 2014. Second, since only post-listing information is used as input to our prediction models and these models have minimum historical data requirements, we excluded companies that have been listed for less than 2 years.

In all databases, there are several companies with missing data. Our dataset is no exception. Excluding those companies with missing data is a source of potential error in evaluating prediction models (Zmijewski, 1984; Platt and Platt, 2012). Therefore, in order to minimise any bias related to this aspect, we only excluded those companies with missing values for the main accounting book items (e.g., sales, total assets) and market information (e.g., price) which are required for computing many accounting and market-based ratios (Lyandres and Zhdanov, 2013). The remaining companies with missing values were dealt with by replacing the missing values for each company by its most recently observed ones (Zhou et al., 2012).

As to outlier values amongst the observed variables, we winsorized these variables; that is, we sat the values lower (respectively, greater) than the 1st (respectively, 99th) percentile of each variable equal to that value (Shumway, 2001).

With respect to the definition of distress, we considered the proposed definition by Pindado et al. (2008). The distress definition is represented by a binary variable, say D, equals 1 for financially distressed companies and equals 0 otherwise, where a company is considered financially distressed if it meets both of the following conditions: (1) its earnings before interest, taxes, depreciation and amortization (EBITDA) is lower than its interest expenses for two consecutive years, and (2) the company experience negative growth in market value for two consecutive years. Details on the number of companies in our dataset and their distress status are provided in Table 2. Notice that the legal aspects of distress complement the financial ones, which strengthens the overall definition of distress given that a relatively low proportion of companies fall under code 21.

Table 1: Basic Sample Statistics

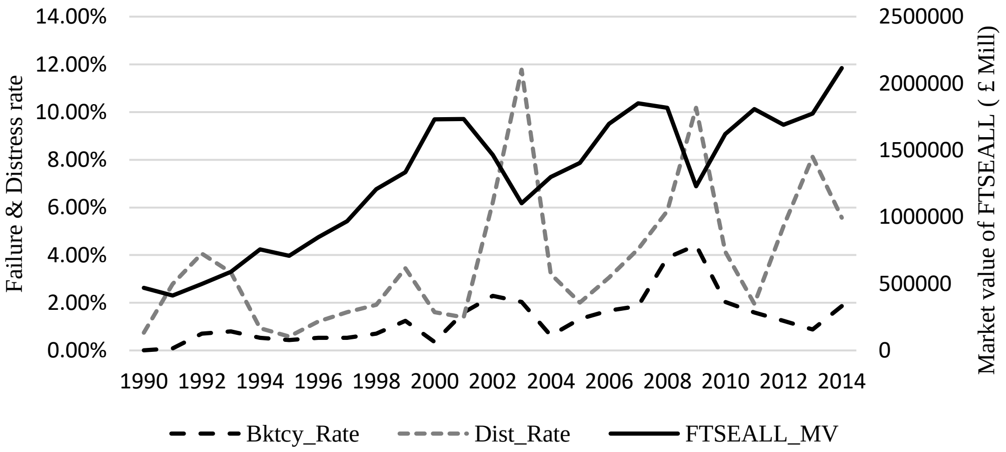
This table presents the total number of distressed companies versus healthy ones for the period of 1990 and 2014.

Observation (1990-2014)	**#**	**%**
Distressed company-year observations (D)	1414	3.82%
Healthy company-year observations	35,570	96.18%
Total company-year Observation	36,984	100%

In sum, our dataset consists of 3,389 companies and 36,984 company-year observations. Among the total number of observation, there are 1,414 company-year observations classified as distressed resulting in a distress rate average of 3.82% per year.

Figure 3 displays the market value of LSE as measured by the FTSE-all index, the average of financial distress and failure rate during 25 years from 1990 through 2014. This graphical snapshot clearly highlights the consistency between our chosen definitions of distress. In addition, the percentage of failed companies as well as our distress variables expressed in percentage terms and the performance of the UK stock market are, as one would expect, inversely moving together in a consistent fashion, which suggest that the use of market information would in principle enhance distress prediction.

Figure 3: Financial distress rate and market value of LSE trend



4.2. Feature Selection

There is a variety of strategies and methods for identifying the most effective group of features to feed failure prediction models with (Balcaen and Ooghe, 2006). Feature selection strategies could be theoretically grounded, empirically grounded, or both – see, for example, Laitinen and Suvara (2016). On the other hand, feature selection methods could be objective or subjective. Objective feature selection methods could be statistical (e.g., Tsai, 2009; Zhou et al., 2012) or non-statistical (e.g., Pacheco et al., 2007, 2009; Unler and Murat, 2010) but adopt a common approach; that is, optimizing an effectiveness criterion. Whereas subjective feature selection methods make often use of a subjective decision rule including reviewing the literature and selecting the most commonly used features (e.g., Ravi Kumar and Ravi, 2007; Zhou, 2013, 2014; du Jardin, 2015; Cleary and Hebb, 2016). In this research, we used a statistical objective feature selection method. To be more specific, we reduced our very large initial set of accounting-based ratios (i.e., 83 accounting-based ratios) to 31 accounting-based features using factor analysis, where factors are selected so that both the absolute values of their loadings are greater than 0.5 and their communities are greater than 0.8, and the stopping criterion is either no improvement in the total explained variance or no more variables are excluded. This factor analysis was run using principal component analysis with VARIMAX as a factor extraction method (Chen, 2011; Mousavi et al., 2015).

4.3. Sample Selection

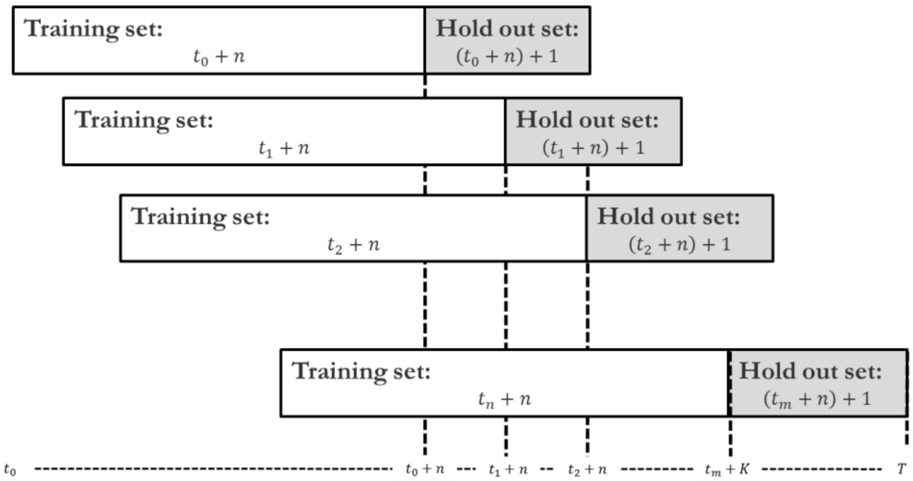
Following the lead of Mousavi et al. (2015), we test the performance of distress prediction models out-of-sample; however, in this paper out-of-sample testing is implemented within a rolling horizon framework. The aim here is to find out how robust is the out-of-sample performance of dynamic distress prediction models relative to static ones with respect to sample type (i.e., pre-crisis, crisis period, post-crisis) and sample period length. In our empirical investigation, we considered three sample period lengths; namely, 3, 5, and 10 years. In sum, we use firm-year observations from year $t-n+1$ to year t ($n=3,5,10$) as a training sample to fit models; that is, estimate their coefficient. Then, we use the fitted models to predict distress in year $t+1$. For the sake of comparing the predictive ability of different models for different samples and different sample period lengths, we are concerned with predicting distress from 2000 onwards; that is, $t=1999$ to 2013. The reader is referred to Figure 4 for a graphical representation of this process. The details about the proportion of distressed firms for each training and holdout sample are presented in Table 2.

Table 2 : The proportion of distress firms (D) in training and holdout samples

This table presents the yearly proportion of distress in our training and hold-out samples. The proportion of distress is presented based on definition of distress (D) and three different length of training period.

Hold out sample		**3-year training sample**		**5-year training sample**		**10-year training sample**	
Year	**D %**	**Years**	**D %**	**Years**	**D %**	**Years**	**D %**
2000	1.60%	1997-1999	2.32%	1995-1999	1.79%	1990-1999	2.04%
2001	1.39%	1998-2000	2.32%	1996-2000	1.96%	1991-2000	2.11%
2002	6.22%	1999-2001	2.15%	1997-2001	1.99%	1992-2001	1.97%
2003	11.78%	2000-2002	3.04%	1998-2002	2.89%	1993-2002	2.23%
2004	3.21%	2001-2003	6.42%	1999-2003	4.82%	1994-2003	3.09%
2005	2.00%	2002-2004	6.97%	2000-2004	4.77%	1995-2004	3.29%
2006	3.06%	2003-2005	5.37%	2001-2005	4.76%	1996-2005	3.38%
2007	4.25%	2004-2006	2.75%	2002-2006	4.99%	1997-2006	3.54%
2008	5.86%	2005-2007	3.13%	2003-2007	4.62%	1998-2007	3.81%
2009	10.18%	2006-2008	4.37%	2004-2008	3.69%	1999-2008	4.21%
2010	4.15%	2007-2009	6.59%	2005-2009	4.94%	2000-2009	4.86%
2011	1.96%	2008-2010	6.77%	2006-2010	5.41%	2001-2010	5.10%
2012	5.21%	2009-2011	5.66%	2007-2011	5.37%	2002-2011	5.18%
2013	8.12%	2010-2012	3.76%	2008-2012	5.63%	2003-2012	5.09%
2014	5.56%	2011-2013	4.99%	2009-2013	6.01%	2004-2013	4.71%

Figure 4: Rolling window periodic sampling



4.4. Distress Prediction Models for Comparative Study

The academic literature includes a broad number of FPMs, which have been employing to predict corporate failure. As mentioned earlier, generally, FPMs could be classified into two main categories; namely, statistical models and non-statistical models. The focus of this study is on statistical models, which could be classified into two sub-categories, namely, static and dynamic models. The selection choice of static models in our comparative analysis is based on two factors; the pioneering proposed static frameworks, and the most frequent applied frameworks in other comparative studies. Further, the selection choice of dynamic models is based on two criteria; the most frequent applied dynamic frameworks in other comparative studies and the recent proposed dynamic frameworks in the literature. As a result of mentioned criteria, we end up with four static frameworks (i.e., univariate discriminant analysis, multivariate discriminant analysis, logit analysis, probit analysis), and four dynamic frameworks (i.e., contingent claim analysis (CCA) models, duration independent hazard models without time-invariant baseline, duration independent hazard with time invariant baseline and duration dependent hazard with time variant baseline).

To be more specific, the traditional accounting based models considered in our comparative study include the univariate discriminant analysis (UDA) model proposed by Beaver (1966); the MDA models proposed by Altman (1968), Altman (1983), Taffler (1983) and Lis (1972); the logit model proposed by Ohlson (1980); the probit model proposed by Zmijewski (1984); and, the linear probability model propose by Theodossiou (1991). The dynamic models considered in our study include; BSM-based models proposed by Hillegeist et al. (2004) and Bharath, Shumway (2008) and the naïve down-and-out call (DOC) barrier option model proposed by Jackson and Wood (2013), duration-independent hazard model with time-invariant baseline proposed by Shumway (2001), duration-independent hazard model without time-invariant baseline model, and duration-dependent with time-variant baseline proposed by Kim and Partington (2014).

4.4.1. Traditional statistical techniques

4.4.1.1. Discriminant analysis (DA)

DA was firstly proposed by Fisher (1938) to classify an observation into two or several a priori categories dependent upon the observation's individual features. The initial objective of DA is to minimize within-group distance and maximize between-group distance (also, referred as Mahalanobis distance). Assuming there are n groups, the generic form of DA model for the group k could be shown as follows;

$$z_k = f\left(\sum_{j=1}^{p} \beta_{kj} x_j\right) \tag{13}$$

where x_j is the discriminant features j, β_{kj} is the discriminant coefficients of group k for discriminant feature j, z_k represents the score of group k, and f is the linear or non-linear classifier that maps the scores, say $\beta^t x$ onto a set of real numbers. Note that to compare DA models to other statistical models, we need to estimate the probability of failure, which is used as an input for estimating many measures of performance. For this, we follow Hillegeist et al. (2004) in using a logit link to calculate the probability of failure for companies;

$$P(failure)_i = \frac{e^z}{1 + e^z} \tag{14}$$

The main shortcoming of DA is that its suitability in optimal discrimination between groups rests on satisfying two underlying assumptions, i.e., the joint normal distribution of features and equal group variance-covariance matrices (Collins and Green, 1982). Although, in practice, the features are rarely normally distributed (Eisenbeis, 1977; Mcleay, 1986) and the groups are hardly equal in variance-covariance matrices (Hamer, 1983), the robustness of DA against deviation from these assumptions for optimality, makes it a widely used method of classification (du Jardin and Séverin, 2012). In this study, we examine the UDA model proposed by Beaver (1966); the MDA models proposed by Altman (1968), Altman (1983), Altman et al. (1995), Taffler (1983) and Lis (1972).

4.4.1.2. Regression Models

The linear probability model (LPM), introduced by Meyer and Pifer (1970) in failure prediction, is a special case of OLS regression when the dependent variable is dichotomous. Similar to OLS, LPM assumes the linear relationship as $Y = X\beta + \epsilon$; where Y is an $N \times 1$ vector of dependent variable, X is an $N \times K$ matrix of independent variables (features), and, ϵ is an $N \times 1$ vector of error terms with $E(\epsilon) = 0$. Generally, in case of failure prediction, Y is a Bernoulli random variable (say, Y equals 1 or 0) and by its very nature two corresponding likelihood, say probability of failure (P) where $Y_i = 1$ and probability of non-failure $(1 - P)$ where $Y_i = 0$ are apparent. Given P and $1 - P$ and binary values of Y, then $E(Y_i) = 1(P) + 0(1 - P)$ which indicates that $X\beta = P$.

One of the major shortcomings of LPM is that ϵ, which equals to $Y - X\beta$, could be either $1 - \beta X$ (if $Y = 1$) or $-\beta X$ (if $Y = 0$) and as such cannot have a normal distribution. Further, since Y is dichotomous variable, and P $[or\ X\beta]$ is constant, the variance of ϵ is same as variance of Y, which is a function of X. Based on Collins and Green (1982), as far as $\hat{Y}$ is in the range of 0.2 and 0.8, this drawback is not serious, although OLS is not an efficient estimator and someone may employ some form of generalized linear square (GLS) to estimate coefficients. Further, practitioner who applying LPM have confidence in their technique, and find it fast and flexible method that provides a very respectable job of predicting failure with ranking comparable to other methodologies (Anderson, 2007; Meyer and Pifer, 1970).

The generic linear probability model (LPM) results in an estimate of probability of failure, the formula for which is as follows;

$$P(failure)_i = \beta_{\text{o}} + \sum_{j=1}^{p} \beta_j x_{ij} \qquad (15)$$

where P is the probability of failure for company i, β_{o} is the constant, β_j is the coefficient of variable x_{ij}.

To overcome some of the restriction of DA and LPM, the probability models of logit and probit were employed in the literature. More specifically, when using a logit or probit framework, the normality and the homoscedasticity assumptions are relaxed. The generic model for binary variables could be stated as follows:

$$\begin{cases} P(failure) = P(Y = 1) \\ P(failure) = G(\beta, X) \end{cases} \tag{16}$$

where Y denotes the binary response variable, X denotes the vector of features, β denotes the vector of coefficients of X in the model, and $G(.)$ is a link function that maps the scores of $\beta^t x$, onto a probability. In practice, depending on choice of link function, the type of probability model is determined. For example, the logit model (respectively, probit model) assumes that the link function is the cumulative logistic distribution; say Θ (respectively, cumulative standard normal distribution, say N) function.

$$G(\beta, X) = \Theta^{-1}(\beta^t X) \tag{17}$$

$$G(\beta, X) = N^{-1}(\beta^t X) \tag{18}$$

In this study, we examine the logit model proposed by Ohlson (1980), the probit model proposed by Zmijewski (1984), and the linear probability model proposed by Theodossiou (1991).

4.4.2. Contingent Claims Analysis (CCA) Models

4.4.2.1. Black Scholes Merton (BSM) Based Models

These models are based on option-pricing theory of Black and Scholes (1973) and Merton (1974), namely BSM, to estimate the probability of failure from market-based information. Before explaining the extraction process of the probability of failure from BSM option pricing theory, it is worthy to consider some points; first, the basic BSM is used to model the price of an option as a function of the underlying stock price and time. Second, a specific type of stochastic process, namely, Itô process, where refers to a Generalized Wiener process with both drift and variance rate, has proven to be a valid framework to model stock prices behaviour for derivatives. Third, purchasing a company's equity can be assumed as taking long position in a call option with an exercise price equal to the face value of its debt liabilities. Based on the mentioned points, as suggested by McDonald (2002) the probability of failure can be extracted as the probability that call option expires worthless at the end of maturity date – i.e. the value of the company's assets (V_a) be less than the face value of its debt liabilities (L) at the end of the holding period, $P(V_a < L)$;

$$P(Failure) = N\left(-\frac{\ln\left(\frac{V_a}{L}\right) + (\mu - \delta - 0.5\sigma_a^2) \times T}{\sigma_a\sqrt{T}}\right) \qquad (19)$$

where $N(.)$ is the cumulative distribution function of the standard Normal distribution, V_a is the value of the company's assets, μ is the expected return of company, σ_a^2 is the volatility of the company's asset, δ is the divided rate, which is estimated by the ratio of dividends to the sum of total liabilities (L) and market value of equity (V_e), L is the total liabilities of the company, and T is time to maturity for both of call option and liabilities. However, to estimate probability of failure in equation 6, someone needs to estimate unobserved parameters of V_a and σ_a.

In proposed approach by Hillegeist et al. (2004), V_a and σ_a are estimated by solving the systems of equations; i.e. the call option equation (20.1) and the optimal hedge equation (20.2).

$$\begin{cases} V_e = V_a e^{-\delta T} N(d_1) - L e^{-rT} N(d_2) + \left(1 - e^{\delta T}\right) N(d_1) V_a & (20.1) \\ \sigma_e = \dfrac{V_a e^{-\delta T} N(d_1) \sigma_a}{V_e} & (20.2) \end{cases} \qquad (20)$$

where V_{e} is the market value of common equity at the time of estimation, σ_{e} is the annualized standard deviation of daily stock returns over 12 months prior to estimation, r is the risk-free interest rate, and d_1 and d_2 are calculated as follows:

$$d_1 = \frac{\ln\left(\frac{Va}{L}\right) + (r - \delta - \frac{1}{2}.\sigma_e^2) \times T}{\sigma_e\sqrt{T}}; \; d_2 = d_1 - \sigma_e\sqrt{T} \qquad (21)$$

Then, μ (expected return of the company) is estimated as follows and is limited between r (risk-free interest rate) and 100%:

$$\mu = \frac{V_{a,t} + D_t - V_{a,t-1}}{V_{a,t-1}} \tag{22}$$

where $V_{a,t}$ is the value of the company's assets in year t and $V_{a,t-1}$ is the value of the company's assets in year $t - 1$.

Alternatively, Bharath and Shumway (2008) proposed a naïve approach to estimate V_a and σ_a as follows:

$$V_a = V_e + D\,; \sigma = \frac{V_e}{V_a}\sigma_e + \frac{D}{V_a}\sigma_d \tag{23}$$

where $\sigma_d = 0.05 + 0.25\sigma_e$. Further, the firm's expected return μ is proxied by the risk-free rate, r or the stock return of previous year restricted to be between r and 100%.

In this study, we apply BSM-based models proposed by Hillegeist et al. (2004) and Bharath, Shumway (2008).

4.4.2.2. Naïve Down-and-Out Call (DOC) Barrier Option Model

In extension to BSM model, the barrier option approach assumes that debt holders' position in a firm is like taking positon in a portfolio of risk-free debt and a DOC option with a strike price equal to a predetermined barrier. This DOC option can be exercised once the value of the company's assets (V_a) is less than the predetermined barrier, B. (For further details about DOC barrier option, the reader is referred to Reisz and Perlich (2007)).

In the naïve DOC barrier option failure prediction model, proposed by Jackson and Wood (2013), the firm's total liabilities is taken as the barrier, B. Therefore, the failure is considered as the status that the value of the company's assets be less than total liabilities, i.e., $V_a < L$. Further, this model rest on the assumptions of no dividends, zero rebate, costless failure proceeding, and set return on asset equal to risk-free rate. The probability of failure using this model is estimated as follows:

$$P(failure) = N\left[\frac{ln\left(\frac{L}{V_a}\right) - \left(\mu - \frac{1}{2}\sigma_e^2\right)T}{\sigma_e\sqrt{T}}\right] + \left(\frac{L}{V_a}\right)^{\frac{2(\mu)}{\sigma_e^2}-1} N\left[\frac{ln\left(\frac{L}{V_a}\right) - \left(\mu - \frac{1}{2}\sigma_e^2\right)T}{\sigma_e\sqrt{T}}\right] \tag{24}$$

In this study, we apply naïve DOC barrier option model in comparative analysis.

4.4.3. Survival Analysis (SA) Models

The survival analysis models are concerned with the analysis of time to event (e.g. failure or distress). Two functions of the interest in survival analysis are the survival function, say $S\ (t)$ and the hazard function, say $h\ (t)$. The survival function, $S\ (t)$ is the probability that the duration of time till the firm faces the event, T, is more than some time t. In other words, $S\ (t)$ can be defined as the probability that the firm survives during the time span of t:

$$S(t) = P(T \geq t) = 1 - F(t) = \int_t^\infty f(u)du \tag{25}$$

where T is the time to failure or the duration of time until firm's event, which is a continuous random variable that follows a probability density function, say $f\ (t)$, and a cumulative density function, $F\ (t)$.

The hazard function or simply hazard, $h\ (t)$, is the immediate rate of event at time $T =\ t$ given the firm survival until the start of the period, which can be defined as follows;

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t \leq T < t + \Delta t | T \geq t)}{\Delta t} = \frac{f(t)}{S(t)} \tag{26}$$

where $h(t)$ is the immediate rate of event at time $T = t$ conditional on the firm survival up to time t.

The key advantage of survival analysis models is that it controls for both the occurrence and the timing of events. While other statistical models estimate the probability of the event using variables based on data at one-time point, survival analysis accommodate changes in the probability of the event due to changes in the values of variables over time (Routledge and Morrisons, 2012).

The pioneer and the most commonly used continues time survival model is the Cox's (1972) semiparametric proportional survival model. The Cox survival model is especially helpful in estimating models with time-varying features. Here, the explanatory variables are entitled to vary in value over the survival period. Therefore, for example, a vector of ratios giving a firm's total debt to assets over a 5-year observation period would be employed as a single variable, but the value of that variable would be renewed as the firm is observed over time in the survival model estimations. The Cox's survival model with time-varying features can be presented as (Andersen, 1992) :

$$h_i(t|x(t)) = \exp\left\{\sum_{j}^{k} \beta_j x_j^i(t)\right\}. h_0(t) \tag{27}$$

where $h_i(t|x(t))$ denotes the time varying hazard function for firm i at time t, $x_j^i(t)$ represents the value of jth explanatory variable of frim i at time t , β_j is the coefficient of the vector x_j^i, and $h_0(t)$ is the baseline hazard function, which represents the effect of time and could be interpreted as the hazard rate with all explanatory variables set to zero. In this structure, the existing hazard depends both on the independent variables (using the $\exp\{\sum_j^k \beta_j x_j^i(t)\}$) and the duration of time the firm has been at risk (using $h_0(t)$). The main advantage of proportional survival models is that it allows for estimation of the parameters of interest (β) in the presence of an unknown, and possibly complicated, time varying baseline hazard (Beck et al., 1998).

In the literature of bankruptcy prediction, two types of survival functions have been employed. The first type of survival function, after taking logit transformation, is a linear function of features (e.g., Chava and Jarrow, 2004; Shumway, 2001). The second type of survival function implements the Cox's proportional survival function (e.g., Allison, 1982; Kim and Partington, 2014). Further, the baseline hazard rate, i.e., the hazard rate when all the features are equal to zero, has a key role in prediction of failure using survival functions. Here, we express a classification of survival analysis models based on a variety of baseline hazard rate and survival function;

4.4.3.1. Duration independent hazard models with (and without) time-invariant baseline

In duration-independent hazard models, the baseline hazard rate could be assumed as a time-invariant (constant) term. For example, Shumway (2001) introduced a discrete-time hazard model using an estimation procedure similar to the one used for estimating the parameters of a multi-period logit model – this choice is motivated by a proposition whereby he proves that a multi-period logit model is equivalent to a discrete-time hazard model. Shumway employed a time-invariant constant term, ln (age), as baseline hazard rate.

Further, a duration- independent hazard model could be assumed without time-invariant baseline. For example, Campbell et al. (2008) employed the suggested discrete time survival models of Shumway, but without time-invariant baseline rate.

4.4.3.2. Duration dependent hazard models with time variant baseline

A duration-dependent type of the baselines is employed in different ways in the literature. Beck et al. (1998) employed time dummies, k_t, representing the length of the sequence of zeros that precede the current observation, as a proxy for the baseline hazard rate. Implementing this type of time dummies as the baseline hazard rate indicates that an individual hazard rate is represented by each firm's survival period.

Nam (2008) argued that employing indirect measure of baseline like time dummies would be not effective proxy in obtaining economy wide condition. This is because the firm’s survival time does not necessarily carry macro-dependencies of firm. Alternatively, some studies proposed employing macroeconomic features like changes in interest rates (Hillegeiste et al., 2001) and the volatility of foreign exchange rate (Name et al., 2008) as the direct measure of baseline hazard rate.

For estimating the probability of failure using the Cox’s proportional survival function, a scaled baseline hazard rate is used which is scaled up, or down based on the firms' risk features. When time-varying features are employed in the Cox model, estimating the baseline hazard rate has been challenging (Chen et al., 2005; Kim and Partington, 2014). As a result, making forecasts has also been infeasible with time-varying features in past financial failure studies. For the first time, Chen et al. (2005) incorporated baseline hazard using Anderson (1992)'s method into a Cox' proportional model to predict the dynamic change of cumulative survival attributed to liver cancer in respect of time-varying biochemical covariates. The integrated baseline survival function can be estimated as follows;

$$\widehat{H}_0(t) = \sum_{\tilde{T}_i \le t} \frac{B_i}{\sum_{j \in R(\tilde{T}_i)} exp\left(\hat{\beta}.x_j(\tilde{T}_i)\right)} \qquad (28)$$

Where B_i is the binary variable for whether the company i experiences the event, i.e. 0 for survivors and 1 for failure or distress; $\tilde{T}_i$ is the event time for the ith company; $x_j(\tilde{T}_i)$ is the value of the jth covariate at the event time of the ith company.

4.5. Performance evaluation of distress prediction models

In this section, firstly, we explain the criteria and measures employed to evaluate the performance of models (see section 4.5.1). Then, we exercise the mono-criteria evaluation of prediction models (see section 4.5.2). Finally, we implement our suggested multi-criteria evaluation approach to evaluate the performance of models (see section 4.5.3).

4.5.1. Criteria and measures for performance evaluation

In this paper, we have focused on the most frequently used criteria and their measures for performance evaluation of prediction models. The first criterion is the discriminatory power, which is defined as the power of a prediction model to discriminate between the healthy firms and the unhealthy firms. In our comparative evaluation, we use H-Measure, Kolmogorov Smirnov (KS), Area under Receivable Operating Characterise (AUROC), Gini index and Information Value (IV) to measure this criterion. The second criterion is the calibration accuracy, which is defined as the quality of estimation of the probability of failure (or distress). We use Brier Score (BS) to measure this criterion. The third criterion is the information content which is defined as the extent to which the outcome of a prediction model (e.g. score or probability of failure) carries enough information for failure (or distress) prediction. We employ log-likelihood statistic (LL) and pseudo-coefficient of determination (pseudo-R^2) to measure this criterion. The last criterion is the correctness of categorical prediction, which is defined as the capability of the failure (or distress) model to correctly classify firms into healthy or non-healthy categories considering the optimal cut-off point. We use Type I errors (T1), Type II errors (T2), misclassification rate (MR), sensitivity (Sen), specificity (Spe), and overall correct classification (OCC) to measure this criterion. – See Appendix C of Mousavi et al. (2014) for descriptions of these measures.

4.5.2. Mono criteria performance evaluation of distress prediction models

In order to answer the first question about the effect of information on the performance of distress models, we employ different combinations of information such as financial accounting (FA), financial accounting and market variables (FAMV), financial accounting and macroeconomic indicators (FAMI), financial accounting, market variables and macroeconomic indicators (FAMVMI), market variables (MV), and market variables and macroeconomic indicators (MVMI) to fed models.

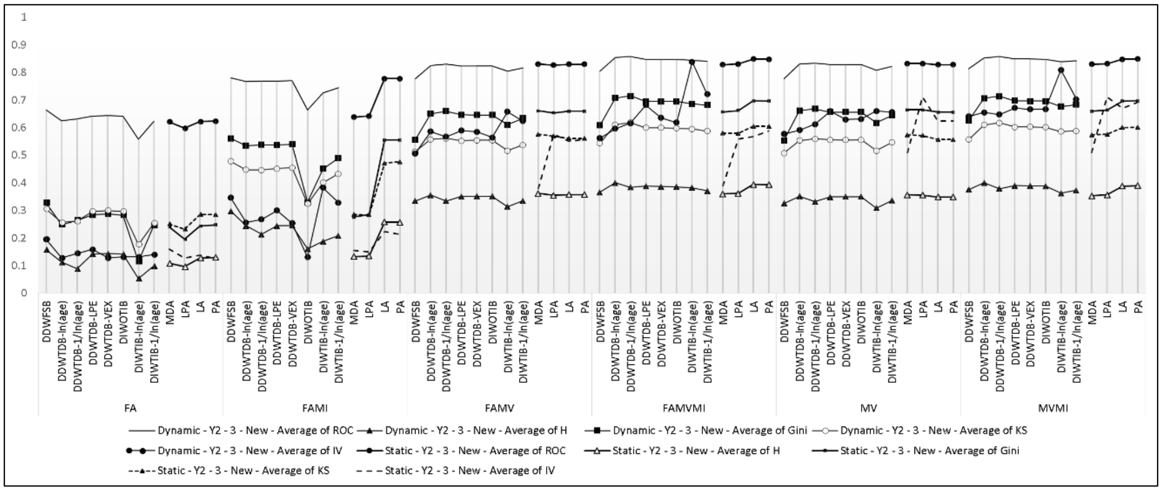
When the availability of information is limited to accounting information (e.g., situations where firms under evaluation are not listed on stock exchanges and macroeconomic information is not available or not reliable), our empirical results demonstrate that accounting information on its own is capable of predicting distressed firms. As one would expect, additional information enhances the ability of all models, whether static or dynamic, to discriminate between firms. In fact, regardless of the performance criterion chosen (i.e., Discriminatory Power, Correctness of Categorical Prediction, Calibration Accuracy) and its measures, empirical results demonstrate that most static and dynamic models perform better when fed with information beyond accounting ones – see, for example, Figure 5, and this enhancement in performance is statistically significant as demonstrated by a substantially large number of one-tailed t-tests of hypotheses involving all combinations of 12 modelling frameworks, 9 categories of information, and 15 measures of 3 performance criteria, where the Null hypothesis H_0 is: Average performance of modelling framework X fed with information category $Y \leq$ Average performance of modelling framework X' fed with information category Y' – see Table 3 for an illustrative example of the typical outcome of these hypothesis tests. In addition, market information (e.g., (log) stock prices, (log) excess returns, volatility of stock returns / unsystematic risk, firm size as proxied by log (number of outstanding shares × year end share price / total market value), its market value, or market value of assets to total liabilities) on its own informs models better than accounting information on its own. However, market and macroeconomic information combined slightly enhance the performance of distress prediction models whether static or dynamic. Furthermore, empirical results suggest that the choice of how a specific piece of information is modelled affects its relevance in adding value to a prediction model. In fact, for example, with respect to the market information category, log(price) is a better modelling choice compared to the price itself and excess return is generally better than log(price).

Table 3: The *p*-value of t-tests to compare the average performance of models using ROC as measure of discriminatory power

This table presents the *p*-value of t-tests to compare the performance of models using ROC. The Null hypothesis (H_0) is: Average performance of modelling framework X fed with information category $Y \leq$ Average performance of modelling framework X' fed with information category Y'.

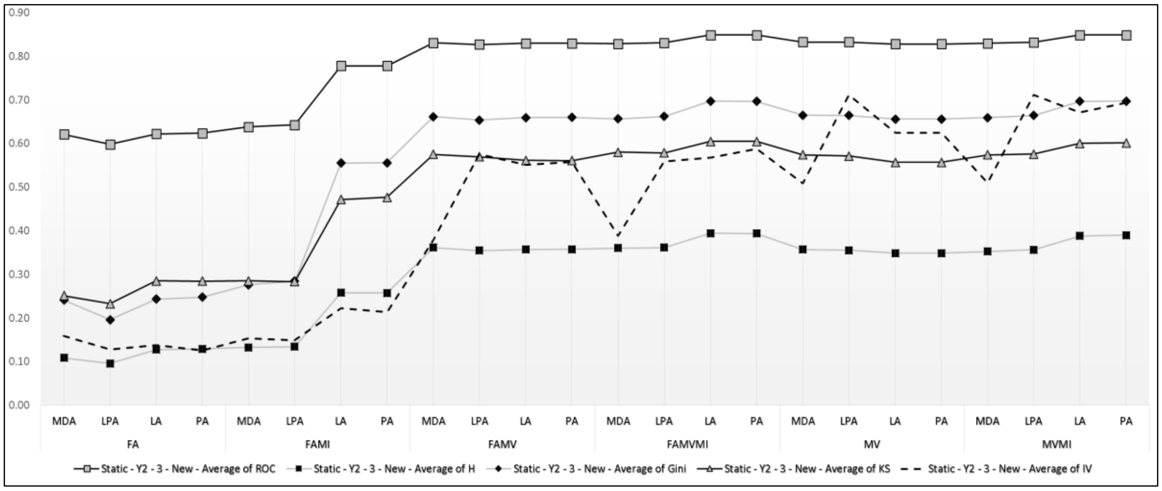
Models	DDWTDB_1/ln(age)_FA	DDWFSB_FAL1MI	DDWFSB_FAMI	DDWFSB_FAMV	DDWFSB_FAMVL1MI	DDWFSB_FAMVMI	DDWFSB_MV	DDWFSB_MVL1MI	DDWFSB_MVMI	DDWTDB_1/ln(age)_FA	DDWTDB_1/ln(age)_FAL1MI	DDWTDB_1/ln(age)_FAMI	DDWTDB_1/ln(age)_FAMV	DDWTDB_1/ln(age)_FAMVL1MI	DDWTDB_1/ln(age)_FAMVMI	DDWTDB_1/ln(age)_MV	DDWTDB_1/ln(age)_MVL1MI	DDWTDB_1/ln(age)_MVMI
DDWFSB_FA		0.007	0.973	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DDWFSB_FAL1MI			1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DDWFSB_FAMI				0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DDWFSB_FAMV					0.145	0.000	0.000	0.000	0.000	0.000	0.000	0.915	0.885	0.000	0.000	0.000	0.000	0.000
DDWFSB_FAMVL1MI						0.000	0.000	0.000	0.000	0.000	0.000	0.942	0.929	0.000	0.000	0.000	0.000	0.000
DDWFSB_FAMVMI							0.001	0.000	0.711	0.004	0.002	1.000	1.000	0.667	0.009	0.006	0.627	0.001
DDWFSB_MV								0.034	0.999	0.978	0.690	1.000	1.000	0.997	0.845	0.663	0.999	0.386
DDWFSB_MVL1MI									0.999	0.997	0.978	1.000	1.000	0.999	0.946	0.857	1.000	0.780
DDWFSB_MVMI										0.001	0.001	1.000	1.000	0.465	0.007	0.005	0.344	0.001
DDWTDB_1/ln(age)_FA											0.036	1.000	1.000	0.995	0.534	0.323	0.998	0.032
DDWTDB_1/ln(age)_FAL1MI												1.000	1.000	0.997	0.795	0.583	0.999	0.233
DDWTDB_1/ln(age)_FAMI													0.359	0.000	0.000	0.000	0.000	0.000
DDWTDB_1/ln(age)_FAMV														0.000	0.000	0.000	0.000	0.000
DDWTDB_1/ln(age)_FAMVL1MI															0.002	0.002	0.422	0.001
DDWTDB_1/ln(age)_FAMVMI																0.071	0.994	0.066
DDWTDB_1/ln(age)_MV																	0.996	0.233
DDWTDB_1/ln(age)_MVL1MI																		0.000
DDWTDB_1/ln(age)_MVMI																		

Figure 5: Measures of Discriminatory Power (ROC, H, Gini, KS, IV) for New Models designed in Different Static and Dynamic Frameworks and Fed with 3-Year Information



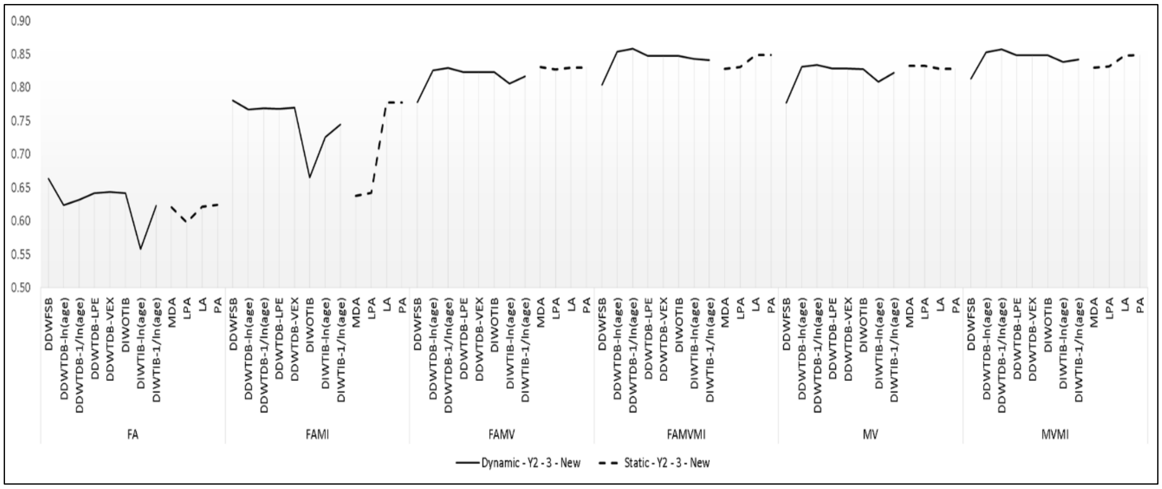
In regards to the second question, which considers the out-of-sample performance of dynamic distress prediction models compare to the out-of-sample performance of static ones with respect to sample type and sample period length, empirical evidence suggests that the out-of-sample implementation of static models within a rolling horizon framework overcomes the a priori limitation of their static nature. In fact, under several combinations of categories of information (e.g., FAMV, FAMVMI, MV), the performance of static models is comparable to the performance of the dynamic ones across all measures of all criteria. This finding suggests that static models are not to be discarded and explains why static models are popular amongst practitioners – see, for example, Figure 7. In addition, the performance of static models is consistent across different combinations of information categories for all measures of all criteria except for information value (IV) and Type I error – see, for example, Figure 5 and Figure 9.

Figure 6: Measures of Discriminatory Power (ROC, H, Gini, KS, IV) of New Static Models Designed in Different Frameworks



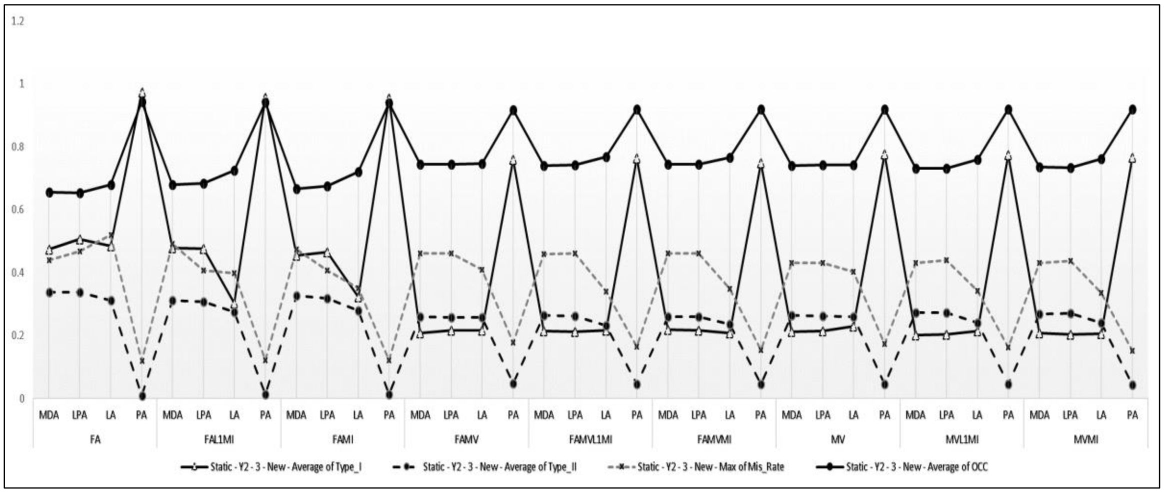
Considering static models and with respect to T1 error, as a measure of correctness of categorical prediction criterion, MDA seems to deliver a higher performance whereas PA is the worst performer. Also, PA performance suggests that this modelling framework is good at properly classifying healthy firms (i.e., it has the smallest Type II error), but relatively speaking, it poorly classifies the distressed ones (i.e., it has the largest Type I error) – see, for example, Figure 9.

Figure 7: ROC of New Models Designed in Different Static and Dynamic Frameworks Fed with 3-year Information



With respect to ROC, H, Gini and KS, as measures of discriminatory power, LA and PA outperform other static models. However, considering IV as measure of discriminatory power, LPA seems to deliver a higher performance whereas MDA is the worst performer – see, for example, Figure 6. However, when static modelling frameworks are fed with both financial accounting and macroeconomic information, there is a clear difference in discriminatory power which suggests that macroeconomic information enhances the performance of LA and PA for discriminatory power measures.

Figure 8: Measures of Correctness of Categorical Prediction of New Static Models Designed in Different Frameworks



With respect to Pseudo-R^2 and Log Likelihood, as measures of information content, LA and PA outperform other static models when fed with accounting and macroeconomic information – see, for example, Figure 10; however, LPA stands out as the best model when market information is used. On the other hand, with respect to measures of quality of fit, such as Brier score, LA outperforms other static models when fed with accounting and macroeconomic information - see, for example,

Much like static models, empirical results suggest that dynamic models perform better when fed with information beyond accounting one; in fact, the performance of dynamic models across most measures of the three criteria under consideration is not only further enhanced when market information is taken on board – see, for example, Figure 5, but it is consistent across all combinations of categories of information that include market variables – see, for example, Figure 5.

With respect to OCC, T2 and MR, as measures of correctness of categorical prediction, DDWTDB_1/ln(age) and DDWTDB_ln(age) [baseline is ignored or equal to 1, and 1/ln(age) and ln(age) are explanatory variables] are the best and second best performers, followed by DIWTIB-1/ln(age) and DIWTIB-ln(age) (1/ln(age) and ln(age) are baselines or intercepts) as average performers, and DDWFSB and DDWTDB-LPE being the worst ones – see, for example, Figure 9. Note however that, with respect to T1, DDWTDB-LPE is the best performer or amongst the best performers regardless of the information categories taken into account. On the other hand, DDWFSB and DDWTDB-VEX are, as expected, being the worst for any combination of information categories that includes market information; however, when market information is not considered, DDWFSB's performance improves while DDWTDB-VEX's performance remains weak – see Figure 9.

With respect to ROC, H, Gini and KS, as measures of discriminatory power, DDWTDB_1/ln(age) is the best performer amongst dynamic models, whereas DDWTDB-ln(age) and DDWFSB are the worst performers – see Figure 5. Once again dynamic models perform better when fed with information beyond accounting ones.

As to information content as measured by Pseudo-R^2 and Log likelihood, DIWTIB_ln(age) (resp. DDWFSB) outperform (underperform) other dynamic models – see, for example, Figure 10.

With respect to the quality of fit, under its BS measure, the performance of DIWTIB_1/ln(age) (resp. DIWTIB_ln(age)) models fed with market variables outperform (resp. underperform) other dynamic models - see, for example, To conclude our comparative analysis of static and dynamic models, we would like to stress out that, in general, static modelling frameworks are as good performers as dynamic ones when implemented under a dynamic scheme. This conclusion suggests that the design of dynamic models along with the information they are fed with need more attention from the academic community for this category of models to perform to the standard it is expected from dynamic frameworks, on one hand, and to become a real contender for practitioners, on one hand.

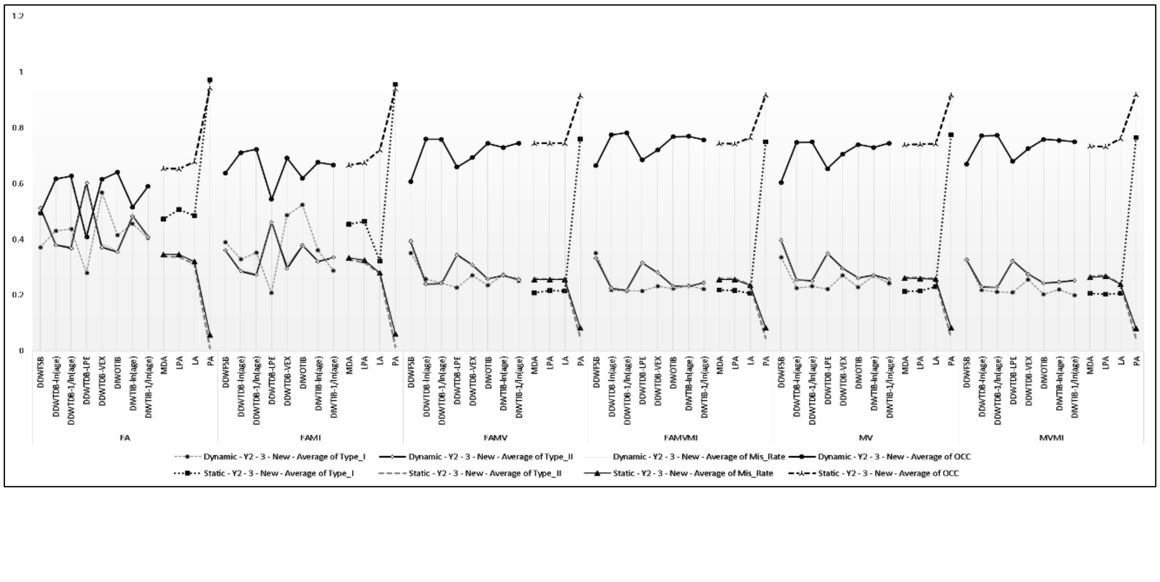
One of the research questions is about the effect of the Length of Training Sample on the performance of models. Under the discriminatory power criterion, a comparison of models under different lengths of the training sample revealed that their empirical performance when market information is taken account of is not significantly affected, except for DDWFSB. In fact, the performance of DDWFSB deteriorates with a longer time window of the training sample – see

Figure **12**. However, when market information is not considered, the performance of models depends to varying extents on the length of the training sample and thus their historical information needs might become lower or higher; e.g., dynamic models fed with 5-year training sample tend to outperform 3-year and 10-year trained models.

Figure 11.

However, feeding dynamic frameworks with information beyond accounting one enhances their calibration accuracy, which suggests that macroeconomic and market information improve the performance of models.

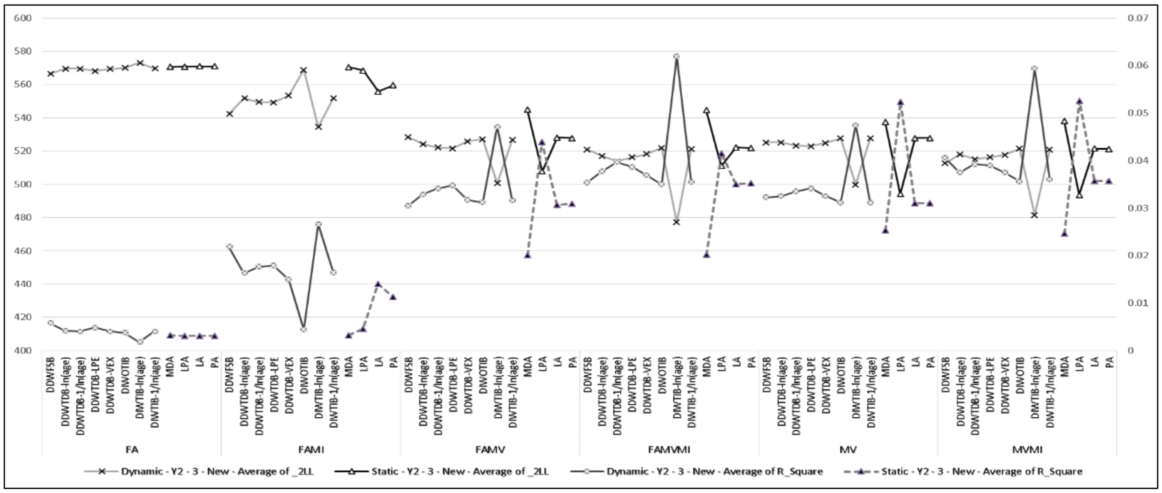
Figure 9: Correctness of Categorical Prediction of New Models Designed in Different Static and Dynamic Frameworks



With respect to both static and dynamic models, under the correctness of categorical prediction criterion, the performance profiles of both static and dynamic models are consistent across different combinations of information categories, however they deliver different performances on different performance measures with the exception of T2 and MR for which both static and dynamic models deliver the same average performance figures – see Figure 9. This latter empirical finding is explained by the fact that MR is a weighted combination of T1 and T2 errors and healthy firms count for the majority of firms in our sample. However, although T1 and OCC are consistent in the way they drive performance, they deliver different figures as expected. One notable behaviour in performance is that of PA being the best performer amongst all static and dynamic models with respect to T2 error, MR and OCC; whereas PA's performance is consistently the worst under T1 errors, on one hand, and the continuous-time hazard model with time-varying baseline based on historical survival period (DDWFSB) is the worst performer across all measures, on the other hand.

Under the discriminatory power criterion, the performance profile of both static and dynamic models are also consistent across different combinations of information categories – see Figure 5. Furthermore, their performance is similar for all measures of discriminatory power except for information value (IV). Generally, DDWTDB_1/ln(age) and LA models are the best performers once the models are fed with market information.

Figure 10: Log likelihood and Pseudo-R^2 of New Dynamic and Static Models Fed with Different Type of Information



As to the calibration accuracy criterion, under measures of both information content and quality of fir, the dynamic model DIWTIB_ln(age) fed with FAMVMI has the highest Pseudo-R^2, the lowest Log Likelihood, and the lowest Brier score; therefore, it outperforms all other models whether static or dynamic – see for Figure 10 and **Error! Not a valid bookmark self-reference.** for example; however, market information boosts LPA models performance to become the best amongst static models.

To conclude our comparative analysis of static and dynamic models, we would like to stress out that, in general, static modelling frameworks are as good performers as dynamic ones when implemented under a dynamic scheme. This conclusion suggests that the design of dynamic models along with the information they are fed with need more attention from the academic community for this category of models to perform to the standard it is expected from dynamic frameworks, on one hand, and to become a real contender for practitioners, on one hand.

One of the research questions is about the effect of the Length of Training Sample on the performance of models. Under the discriminatory power criterion, a comparison of models under different lengths of the training sample revealed that their empirical performance when market information is taken account of is not significantly affected, except for DDWFSB. In fact, the performance of DDWFSB deteriorates with a longer time window of the training sample – see

Figure **12**. However, when market information is not considered, the performance of models depends to varying extents on the length of the training sample and thus their historical information needs might become lower or higher; e.g., dynamic models fed with 5-year training sample tend to outperform 3-year and 10-year trained models.

Figure 11: Brier Score of new dynamic and static models fed with different type of information

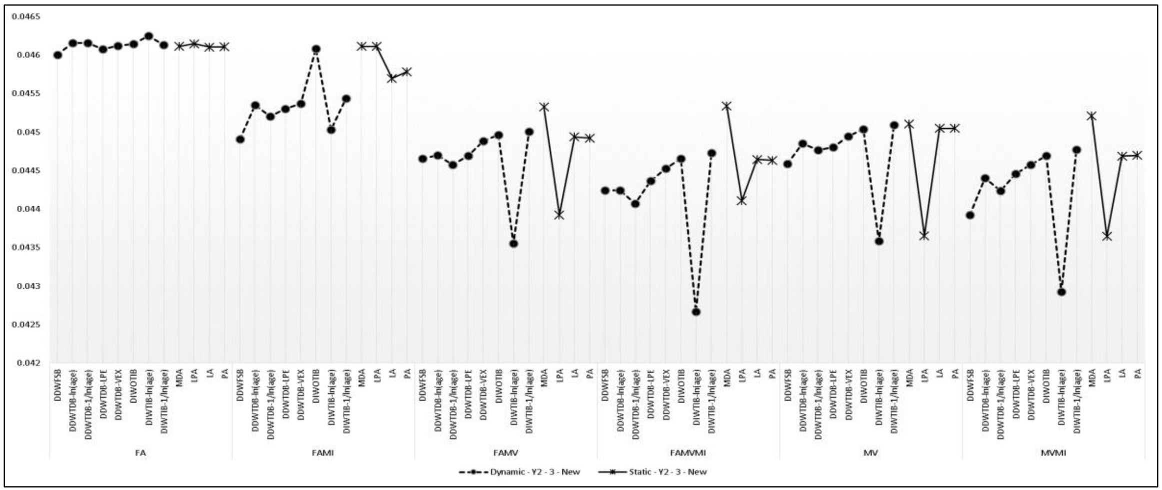
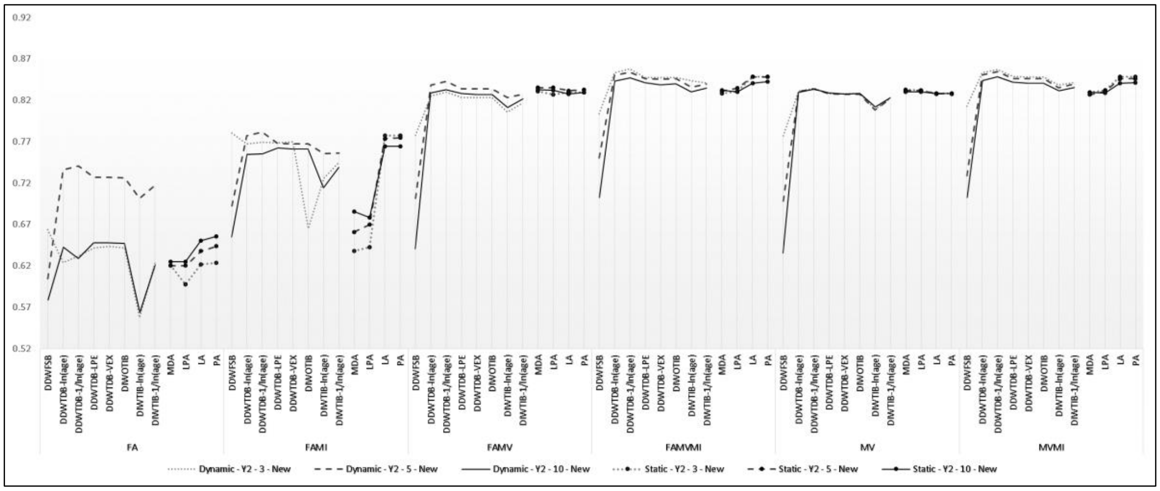


Figure 12: ROC of New Models Fed with Different Length and Type of Information



Under the correctness of categorical prediction criterion, a longer time window of the training sample improves the performance of both static and dynamic models under T1 – see Figure 13. However, under T2, MR and OCC, a shorter time window of the training sample improves the performance of both static and dynamic models – see, Figure 14. In sum, under T1, both static and dynamic modelling frameworks require more historical information than what is required under T2, MR and OCC for a good performance.

Figure 13: Type I Error of New Models Fed with Different Length and Type of Information

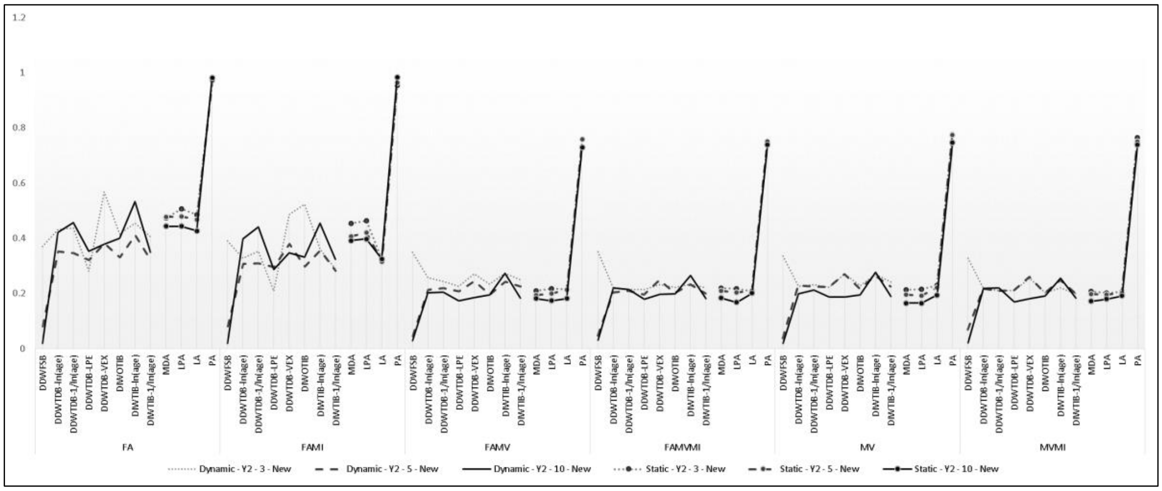
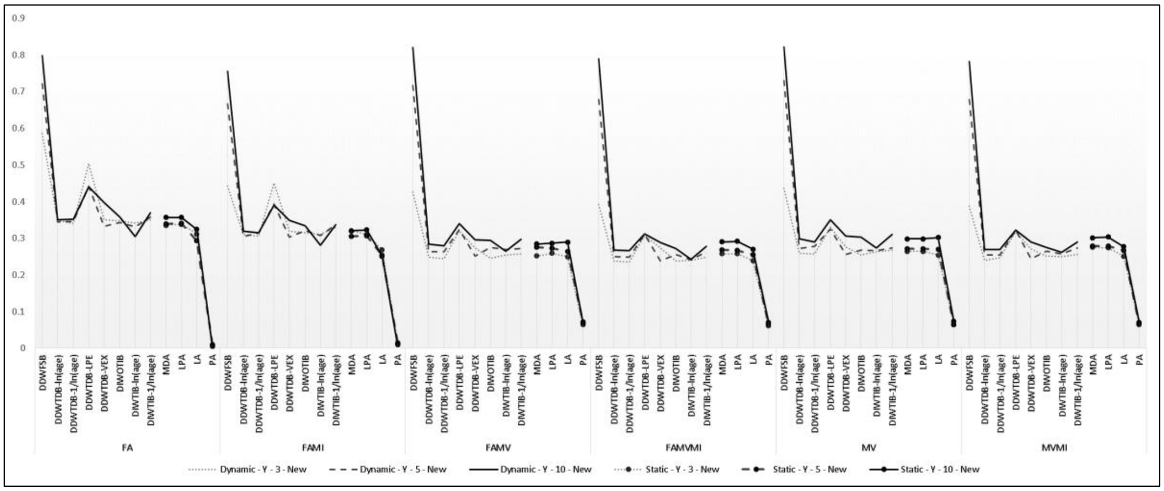
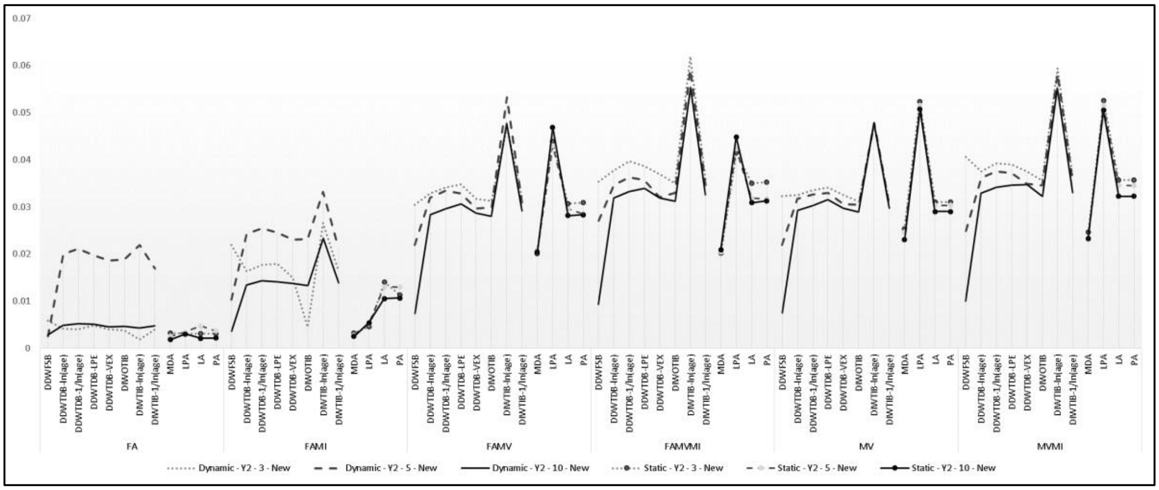


Figure 14: Type II Error of New Models Fed with Different Length and Type of Information



Under the information content criterion and its measures; namely, Pseudo-R^2 and Log likelihood, most models fed with 5-year training sample outperform other models when market information is ignored. However, when market variables are taken into account, a shorter time window of the training sample improves the performance of both static and dynamic models – see, for example, Figure 15.

Figure 15: Pseudo-R^2 of New Models Fed with Different Length and Type of Information

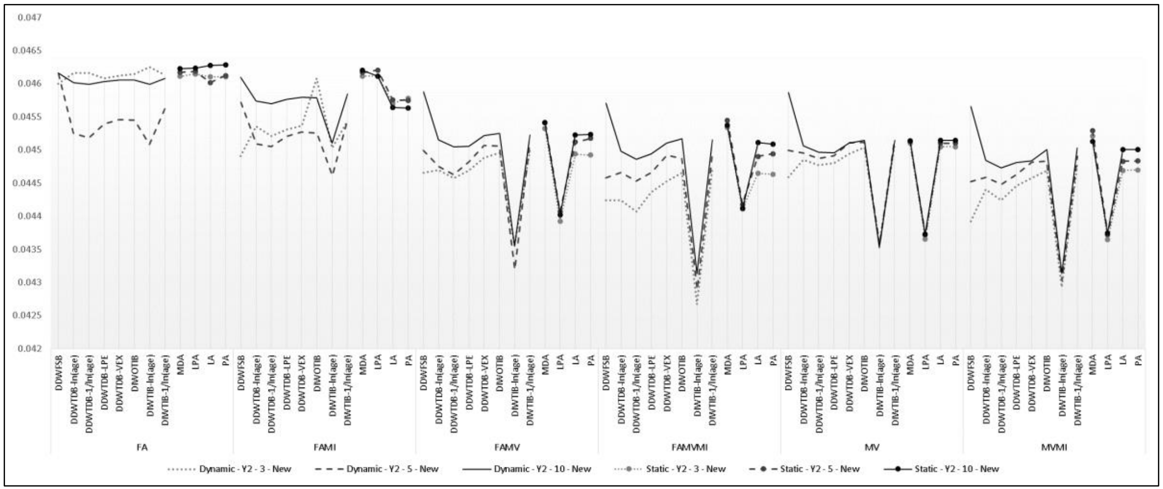


Under Brier score, as a calibration accuracy measure, models fed with 5-year training sample perform better when market information is ignored. However, when static and dynamic models are fed with market information, a shorter time window of the training sample improves their performance – see As suggested by one-dimensional ranking of distress prediction models, taking into account different performance criteria and measures, there are considerable conflicts and ties in ranking of models. Therefore, taking into account multiple criteria, one cannot make an informed decision as to which model performs best. Although, we insist that one-dimensional rankings are not to be discarded, we would like to propose a dynamic multi-criteria assessment, which provides a single ranking under multiple criteria.

Figure 16. In sum, although accounting, market, and macroeconomic information are correlated to varying degrees over time, market information proved to be the most informative prediction-wise.

As suggested by one-dimensional ranking of distress prediction models, taking into account different performance criteria and measures, there are considerable conflicts and ties in ranking of models. Therefore, taking into account multiple criteria, one cannot make an informed decision as to which model performs best. Although, we insist that one-dimensional rankings are not to be discarded, we would like to propose a dynamic multi-criteria assessment, which provides a single ranking under multiple criteria.

Figure 16: Brier Score of New Models Fed with Different Length and Type of Information



4.5.3. Multi criteria performance evaluation of distress prediction models

In this study we considered 12 forecasting frameworks (i.e., MDA, LPA, LA, PA, DDWFSB, DDWTDB_ln(age), DDWTDB_1/ln(age), DDWTDB_LPE, DDWTDB_VEX, DDWOTIB, DDWTIB_ln(age), and DDWTIB_1/ln(age)) which are fed with six groups of information (i.e., FA, FAMI, FAMV, FAMVMI, MV, and MVMI) using three different training periods (i.e., 3, 5 and 10-year information). Due to the superiority of models fed with FAMVMI in mono-criteria rankings and also to save space, we only present the multi-criteria performance evaluation of models fed with FAMVMI. Note however that the same findings are reached under other combinations of information categories.

Setup 1 - Inputs: T1, BS and outputs: Pseudo-R^2, ROC

In the first round of multi-criteria assessment using Malmquist DEA, we used T1 error (as a measure of correctness of categorical prediction) and Brier score (as a measure of quality of fit) as inputs, and Pseudo-R^2 (as a measure of information content) and ROC (as a measure of discriminatory power) as outputs.

Table 4, Table 5 and Table 6 provide the rankings of models based on the estimated efficiency scores during period 2000 to 2014. For easier comparison, we provide a point which is calculated based on the ranking of models over 15-year period.

With respect to the performance of models fed with 3-year training sample, DDWTDB-1/ln(age) outperforms other models, following by DDWTDB-ln(age) and DIWTIB-ln(age) which are the second and third best performers. Note that, although multi-criteria ranking of models suggest that dynamic models are superior to the static ones, LA performs better than the other static models and some dynamic ones.

Considering models fed with either 5 or 10-year training samples, DDWTDB-1/ln(age), DDWFSB and DDWTDB-ln(age) are the best performers. Furthermore, the static model LA outperforms the remaining static models. As the results of mono-criterion ranking suggest increasing the length of the training sample improves the performance of DDWFSB under T1 error. In consistent to mono-criterion, the results of multi-criteria ranking suggest that increasing the period of training sample improves the performance of DDWFSB.

Comparing all models fed with 3, 5 and 10-year training sample, dynamic models perform much better than static models. DDWFSB fed with 5-year information has the best performance over 15-year period, following by other duration dependent models which use ln(age) or 1/ln(age) as baseline rate_ see Table 10 .

Setup 2 - Inputs: T2, BS and outputs: Pseudo-R^2, ROC

In the second round of multi-criteria assessment using Malmquist, we used Type II error (as measure of correctness of categorical prediction) and Brier score (as measure of quality of fit) as inputs, and R^2 (as measure of information content) and ROC (as measure of discriminatory power) as outputs.

Table 7, Table 8 and Table 9 represent the ranking of models in each year based on the estimated efficiency scores using Malmquist DEA. Conversely to the last round of multi-criteria assessment where the static models underperform dynamic ones, the second round of assessment indicates that

PA outperforms other models. With respect to all length of training samples, the PA model outperforms other models. However, the dynamic model of DDWTDB_1/ln(age) is ranked second followed by DDWTDB_ln(age) with respect to all length of training samples.

With respect to all models fed with FAMVMI, the static model of PA fed with 5,3 and 10-year information outperform other models over 15-year period. DDWTDB_1/ln(age) fed with 3,5-year training sample is the second best performer, see Table 11.

Table 4: The first round of multi-criteria ranking of models fed with 3-year FAMVMI information

Framework	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Rank	Rank	Rank	Point
DDWFSB	3	9	3	11	1	4	4	12	10	6	12	2	1	10	3	**2**	**1**	**3**	**89**
DDWTDB-1/ln(age)	1	1	5	2	3	1	2	1	1	2	3	1	2	1	5	**7**	**4**	**2**	**149**
DDWTDB-ln(age)	4	2	6	4	4	2	3	3	3	3	4	5	4	2	7	**0**	**3**	**4**	**124**
DDWTDB-LPE	8	3	9	9	2	8	9	5	5	1	1	3	7	4	1	**3**	**1**	**2**	**105**
DDWTDB-VEX	6	5	1	3	8	5	1	7	4	5	2	6	3	6	8	**2**	**1**	**2**	**110**
DIWOTIB	5	6	8	6	9	9	8	6	6	8	5	4	5	3	6	**0**	**0**	**1**	**86**
DIWTIB-1/ln(age)	9	8	10	7	12	10	10	9	8	9	7	7	8	7	4	**0**	**0**	**0**	**55**
DIWTIB-ln(age)	2	7	2	1	6	3	5	2	2	4	6	8	6	5	10	**1**	**4**	**1**	**111**
LA	7	4	4	5	5	6	6	4	7	7	8	9	9	8	2	**0**	**1**	**0**	**89**
LPA	12	11	11	10	10	11	12	10	11	11	10	11	11	11	11	**0**	**0**	**0**	**17**
MDA	11	12	12	12	11	12	11	11	12	12	11	12	12	12	12	**0**	**0**	**0**	**5**
PA	10	10	7	8	7	7	7	8	9	10	9	10	10	9	9	**0**	**0**	**0**	**50**

Table 5: The first round of multi-criteria ranking of models fed with 5-year FAMVMI information

Framework	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Rank 1	Rank 2	Rank 3	Point
DDWFSB	2	2	5	5	4	3	1	1	8	1	7	1	1	1	1	**7**	**2**	**1**	**137**
DDWTDB-1/ln(age)	4	1	2	1	2	1	2	3	1	2	2	6	2	2	3	**4**	**7**	**2**	**146**
DDWTDB-ln(age)	5	7	3	2	3	4	4	4	2	3	4	9	3	3	5	**0**	**2**	**5**	**119**
DDWTDB-LPE	1	4	7	9	1	7	8	7	6	6	1	3	8	5	4	**3**	**0**	**1**	**103**
DDWTDB-VEX	7	9	4	6	5	5	6	5	7	5	6	4	6	9	6	**0**	**0**	**0**	**90**
DIWOTIB	8	3	6	4	8	6	7	8	5	8	5	5	7	6	7	**0**	**0**	**1**	**87**
DIWTIB-1/ln(age)	10	6	9	7	10	9	10	9	9	9	9	8	9	8	9	**0**	**0**	**0**	**49**
DIWTIB-ln(age)	3	5	1	3	6	2	3	2	4	4	10	10	4	10	10	**1**	**2**	**3**	**103**
LA	6	8	8	8	7	8	5	6	3	7	3	2	5	4	2	**0**	**2**	**2**	**98**
LPA	12	10	11	11	11	11	11	11	12	11	11	11	11	11	11	**0**	**0**	**0**	**14**
MDA	11	12	12	12	12	12	12	12	11	12	12	12	12	12	12	**0**	**0**	**0**	**2**
PA	9	11	10	10	9	10	9	10	10	10	8	7	10	7	8	**0**	**0**	**0**	**42**

Table 6: The first round of multi -criteria ranking of models fed with 10-year FAMVMI information

Framework	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Rank	Rank	Rank	Point
DDWFSB	1	12	9	1	4	3	3	9	1	1	1	1	1	1	1	**9**	**0**	**2**	**131**
DDWTDB-1/ln(age)	2	1	3	3	1	1	2	2	3	2	6	7	2	2	3	**3**	**6**	**4**	**140**
DDWTDB-ln(age)	4	4	4	6	3	4	4	4	5	3	7	9	4	3	4	**0**	**0**	**3**	**112**
DDWTDB-LPE	6	9	8	4	2	5	6	6	4	4	2	5	8	4	2	**0**	**3**	**0**	**105**
DDWTDB-VEX	7	6	2	5	5	6	8	1	2	5	9	4	7	7	6	**1**	**2**	**0**	**100**
DIWOTIB	8	7	7	8	7	7	7	7	8	7	5	2	6	5	5	**0**	**1**	**0**	**84**
DIWTIB-1/ln(age)	10	5	5	9	10	9	10	8	9	9	4	6	9	9	8	**0**	**0**	**0**	**60**
DIWTIB-ln(age)	3	2	1	2	6	2	1	3	7	6	10	11	3	8	11	**2**	**3**	**3**	**104**
LA	5	8	6	7	9	8	5	5	6	8	3	3	5	6	7	**0**	**0**	**2**	**89**
LPA	12	3	11	11	11	11	12	11	11	11	11	10	11	11	10	**0**	**0**	**1**	**23**
MDA	11	11	12	12	12	12	11	12	12	12	12	12	12	12	12	**0**	**0**	**0**	**3**
PA	9	10	10	10	8	10	9	10	10	10	8	8	10	10	9	**0**	**0**	**0**	**39**

Table 7: The second round of multi-criteria ranking of models fed with 3-year FAMVMI information

Framework	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Rank 1	Rank 2	Rank 3	Point
DDWFSB	3	10	8	3	1	5	5	1	10	10	2	8	10	11	10	**2**	**1**	**2**	**83**
DDWTDB-1/ln(age)	4	3	3	5	4	1	1	3	2	2	3	3	3	3	3	**2**	**2**	**8**	**137**
DDWTDB-ln(age)	6	7	5	4	5	3	2	4	3	3	5	6	5	5	5	**0**	**1**	**3**	**112**
DDWTDB-LPE	7	9	6	1	11	9	8	8	7	9	10	5	4	8	8	**1**	**0**	**0**	**70**
DDWTDB-VEX	5	5	9	6	10	8	6	7	8	6	9	2	2	2	2	**0**	**4**	**0**	**93**
DIWOTIB	8	6	7	8	9	7	9	9	6	5	4	4	7	7	6	**0**	**0**	**0**	**78**
DIWTIB-1/ln(age)	10	8	10	10	12	10	10	10	9	7	6	7	9	6	7	**0**	**0**	**0**	**49**
DIWTIB-ln(age)	2	2	2	7	6	4	4	5	4	4	7	9	6	9	9	**0**	**3**	**0**	**100**
LA	9	4	4	9	3	6	7	6	5	8	8	10	8	4	4	**0**	**0**	**1**	**85**
PA	1	1	1	2	2	2	3	2	1	1	1	1	1	1	1	**10**	**4**	**1**	**159**
LPA	12	11	11	11	8	11	12	11	11	11	11	11	11	10	11	**0**	**0**	**0**	**17**
MDA	11	12	12	12	7	12	11	12	12	12	12	12	12	12	12	**0**	**0**	**0**	**7**

Table 8: The second round of multi-criteria ranking of models fed with 5-year FAMVMI information

Framework	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Rank 1	Rank 2	Rank 3	Point
DDWFSB	10	10	10	12	12	12	2	12	12	10	12	10	9	11	10	**0**	**1**	**0**	**26**
DDWTDB-1/ln(age)	3	3	3	3	2	2	4	2	2	2	3	6	3	3	2	**0**	**6**	**7**	**137**
DDWTDB-ln(age)	4	4	4	6	3	3	5	4	3	3	6	8	4	4	7	**0**	**0**	**4**	**112**
DDWTDB-LPE	9	8	6	2	11	5	6	6	5	8	10	3	5	5	6	**0**	**1**	**1**	**85**
DDWTDB-VEX	6	6	9	4	6	7	8	8	6	7	4	2	2	2	4	**0**	**3**	**0**	**99**
DIWOTIB	7	7	7	9	7	8	9	7	8	6	5	5	7	6	5	**0**	**0**	**0**	**77**
DIWTIB-1/ln(age)	8	9	8	8	10	9	10	9	9	9	7	7	10	8	8	**0**	**0**	**0**	**51**
DIWTIB-ln(age)	2	2	2	5	5	4	3	3	4	4	8	9	8	10	11	**0**	**3**	**2**	**100**
LA	5	5	5	7	4	6	7	5	7	5	2	4	6	7	3	**0**	**1**	**1**	**102**
PA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**15**	**0**	**0**	**165**
LPA	12	11	11	10	8	11	11	10	10	11	9	11	11	9	9	**0**	**0**	**0**	**26**
MDA	11	12	12	11	9	10	12	11	11	12	11	12	12	12	12	**0**	**0**	**0**	**10**

Table 9: The second round of multi-criteria ranking of models fed with 10-year FAMVMI information

Framework	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	Rank 1	Rank 2	Rank 3	Point
DDWFSB	11	12	12	12	12	11	9	12	12	12	12	12	12	12	12	**0**	**0**	**0**	**5**
DDWTDB-1/ln(age)	2	2	3	2	2	3	3	2	2	2	2	6	2	2	2	**0**	**11**	**3**	**143**
DDWTDB-ln(age)	4	5	4	4	3	4	4	4	4	3	4	8	3	4	4	**0**	**0**	**3**	**118**
DDWTDB-LPE	8	8	5	9	11	5	6	7	7	8	7	3	4	6	7	**0**	**0**	**1**	**79**
DDWTDB-VEX	6	7	8	5	5	7	7	8	8	7	9	2	6	3	3	**0**	**1**	**2**	**89**
DIWOTIB	7	9	6	7	6	6	8	6	6	6	3	4	8	7	5	**0**	**0**	**1**	**86**
DIWTIB-1/ln(age)	9	10	7	8	8	9	10	9	9	9	6	7	9	9	8	**0**	**0**	**0**	**53**
DIWTIB-ln(age)	3	3	2	3	4	2	2	3	3	4	8	9	5	8	9	**0**	**3**	**5**	**112**
LA	5	6	9	6	7	8	5	5	5	5	5	5	7	5	6	**0**	**0**	**0**	**91**
PA	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	**15**	**0**	**0**	**165**
LPA	12	4	10	10	9	10	12	10	10	10	10	10	10	10	10	**0**	**0**	**0**	**33**
MDA	10	11	11	11	10	12	11	11	11	11	11	11	11	11	11	**0**	**0**	**0**	**16**

5. Conclusion

Prediction of corporate distress is crucial for many decision makers in finance and investment. Although a large number of models have been designed to predict bankruptcy and distress, the relative performance evaluation of competing distress models remains an exercise that is one-dimensional in nature, which results in conflicting rankings of models from one performance criterion to another. In this study, we proposed an orientation-free super-efficiency Malmquist DEA, which provides a single ranking, based on multiple performance criteria. In addition, we exercised a comprehensive comparative analysis of the most famous static and dynamic distress prediction models. For this, we used several measures under four commonly employed criteria (i.e., the discriminatory power, the information content, the calibration accuracy, and the correctness of categorical prediction) in the literature. Furthermore, we addressed the following important questions: What category of information or combination of categories of information enhances the predictive ability of models best? and How the out-of-sample performance of dynamic distress prediction models compare to the out-of-sample performance of static ones with respect to sample type and sample period length?

Our main findings could be classified as follows. Firstly, the proposed multi-criteria dynamic framework provides a useful tool in evaluating the relative performance of distress prediction models over time. Secondly, conversely to the one-dimensional ranking, the multidimensional ranking of models provides more consistent results. However, in case of a significant inconsistency between rankings of a model using Type I and Type II errors (i.e. PA model), multi-criteria rankings of a model using each of these two measures would also present inconsistency. Third, empirical results suggest that dynamic models, specifically DDWTDB_1/ln(age) and DDEWTDB_ln(age) are always amongst the best distress prediction models and show consistency in multi-criteria ranking using different combinations of measures. Forth, models fed with shorter training sample period (i.e. 3-year training period) length are superior in performance. Fifth, most modelling frameworks show improvement in performance by taking account of features beyond accounting-based information (i.e. FAMVMI and FAMV).

Table 10: The first round of multi-criteria ranking of models fed with FAMVMI information

Framework	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**Total rank**
DDWFSB_05	5	1	14	14	13	9	4	1	17	1	19	1	2	12	1	**1**
DDWTDB_1/ln(age)_03	1	15	25	17	14	1	2	3	7	4	11	3	4	2	7	**2**
DDWFSB_10	9	36	13	1	7	8	14	24	1	2	2	2	1	1	2	**3**
DDWTDB_1/ln(age)_05	7	1	6	9	8	1	12	7	2	7	10	20	11	20	13	**4**
DDWTDB_1/ln(age)_10	15	1	4	3	1	5	13	6	13	8	9	14	12	10	23	**5**
DDWTDB_ln(age)_03	8	17	26	21	18	4	3	9	10	5	15	11	6	3	9	**6**
DDWTDB_ln(age)_10	16	10	5	6	5	10	16	11	16	12	12	21	16	13	24	**7**
DDWTDB_ln(age)_05	10	12	7	10	11	11	19	8	3	9	14	27	14	23	15	**7**
DIWTIB_ln(age)_03	1	25	21	16	25	6	6	4	8	6	20	25	8	6	19	**9**
DDWTDB_LPE_03	14	19	29	29	6	23	11	17	21	3	3	5	9	5	3	**10**
DDWTDB_VEX_03	12	23	20	18	28	13	1	22	15	11	4	13	5	7	10	**11**
DDWTDB_LPE_10	20	20	12	4	1	16	20	16	14	13	5	9	26	15	22	**12**
DIWTIB_ln(age)_05	6	8	3	12	20	1	17	5	5	10	26	31	18	31	26	**13**
DIWTIB_ln(age)_10	16	1	1	2	10	7	9	10	20	17	28	29	15	21	34	**14**
DDWTDB_VEX_10	21	13	2	5	9	18	23	2	12	15	25	8	24	19	27	**15**
DDWTDB_LPE_05	1	7	16	24	4	17	30	18	9	16	1	16	25	26	14	**16**
LA_10	19	18	10	7	16	25	18	15	19	19	6	7	19	17	28	**17**
DIWOTIB_10	22	16	11	8	12	20	21	21	22	18	8	6	21	16	25	**18**
DDWFSB_03	1	27	22	34	1	12	5	36	33	20	36	4	3	14	5	**19**
LA_05	23	14	17	20	21	21	22	14	4	21	13	15	20	24	12	**20**
DIWOTIB_03	11	24	28	26	29	24	10	19	24	23	17	10	7	4	8	**21**
DDWTDB_VEX_05	27	21	8	15	17	14	28	12	11	14	18	17	22	30	16	**22**
LA_03	13	22	24	23	23	19	7	13	26	22	27	28	13	9	4	**23**
DIWOTIB_05	28	6	15	13	22	15	29	20	6	24	16	18	23	27	17	**24**
DIWTIB_1/ln(age)_10	26	11	9	11	19	28	27	23	23	25	7	12	27	22	29	**25**
DIWTIB_1/ln(age)_03	18	26	30	27	36	30	15	28	29	27	21	23	10	8	6	**26**
PA_03	24	31	27	28	26	22	8	25	31	32	31	30	17	11	11	**27**
DIWTIB_1/ln(age)_05	30	9	19	19	30	26	32	27	18	26	24	26	29	29	20	**28**
PA_10	25	30	18	22	15	29	24	26	27	28	22	19	28	25	30	**29**
PA_05	29	29	23	25	24	27	31	29	25	31	23	22	30	28	18	**30**
LPA_10	36	5	31	30	27	34	35	30	32	29	30	24	35	32	33	**31**
LPA_03	33	32	33	33	34	31	26	34	35	35	34	35	31	18	21	**32**
LPA_05	34	28	35	32	31	33	33	32	30	33	29	33	32	34	32	**33**
MDA_10	35	34	32	31	32	35	34	31	34	30	32	32	36	35	35	**34**
MDA_03	32	33	34	35	35	32	25	35	36	36	35	36	33	33	31	**35**
MDA_05	31	35	36	36	33	36	36	33	28	34	33	34	34	36	36	**36**

Table 11: The second round of multi-criteria ranking of models fed with FAMVMI information

Framework	**2000**	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**	**Total rank**
PA_05	6	1	3	4	2	1	4	3	3	2	1	1	3	2	1	**1**
PA_03	1	7	1	2	4	4	5	2	1	1	3	3	1	1	2	**2**
PA_10	1	1	2	3	3	5	3	4	2	3	2	2	2	3	3	**3**
DDWTDB_1/ln(age)_03	5	16	21	16	13	1	1	6	4	4	5	6	5	6	5	**4**
DDWTDB_1/ln(age)_05	15	10	8	9	7	1	16	5	5	7	8	20	13	15	9	**5**
DDWTDB_ln(age)_03	9	24	23	13	17	7	2	8	6	5	10	13	7	8	7	**6**
DDWTDB_1/ln(age)_10	11	1	6	7	5	12	13	11	10	8	13	18	14	13	19	**7**
DIWTIB_ln(age)_03	1	8	20	25	20	8	6	10	7	6	19	24	8	12	17	**8**
DDWTDB_ln(age)_05	18	13	10	14	9	6	18	9	8	10	12	25	16	18	15	**9**
DDWTDB_ln(age)_10	13	9	7	11	6	13	17	13	12	11	15	23	15	16	22	**10**
DIWTIB_ln(age)_10	13	1	5	8	8	10	10	12	11	12	23	27	20	25	31	**11**
DIWTIB_ln(age)_05	8	1	4	12	12	9	12	7	9	13	25	28	23	31	30	**12**
DDWTDB_VEX_03	7	22	27	20	31	18	8	16	25	14	24	4	4	5	4	**13**
DDWFSB_03	1	29	26	5	1	11	7	1	32	28	4	22	18	28	21	**14**
DIWOTIB_03	12	23	25	26	29	17	15	20	23	9	6	7	9	10	8	**15**
DDWTDB_VEX_05	24	18	19	10	15	20	29	27	14	21	9	5	12	4	12	**15**
LA_03	17	19	22	27	11	14	11	14	21	17	21	26	10	7	6	**17**
LA_05	22	17	12	18	10	16	25	21	15	15	7	11	21	24	11	**18**
DDWTDB_LPE_03	10	28	24	1	33	21	14	18	24	25	27	12	6	11	16	**19**
LA_10	16	11	18	17	19	25	19	15	17	18	18	16	26	17	24	**20**
DIWOTIB_10	21	15	11	19	16	22	23	17	18	20	14	14	27	21	23	**21**
DDWTDB_LPE_05	28	21	13	6	32	15	24	22	13	22	28	9	17	19	14	**22**
DDWTDB_VEX_10	20	12	17	15	14	24	22	25	22	23	29	8	24	14	20	**23**
DDWTDB_LPE_10	23	14	9	23	28	19	20	19	20	24	22	10	19	20	25	**24**
DIWOTIB_05	25	20	14	24	18	23	31	24	16	19	11	15	22	22	13	**25**
DIWTIB_1/ln(age)_03	19	27	28	28	34	28	21	23	27	16	16	17	11	9	10	**26**
DIWTIB_1/ln(age)_05	27	26	15	22	30	27	32	28	19	27	17	21	28	26	18	**27**
DIWTIB_1/ln(age)_10	26	25	16	21	22	26	30	26	26	26	20	19	29	27	26	**28**
LPA_10	36	6	31	29	25	31	34	29	30	29	32	30	34	30	33	**29**
DDWFSB_05	29	31	29	35	35	36	9	35	31	30	35	29	25	34	29	**30**
LPA_03	31	35	30	32	24	29	27	33	34	35	30	35	30	23	27	**31**
LPA_05	33	32	34	31	21	35	35	30	28	32	26	32	32	29	28	**32**
MDA_10	34	30	33	30	26	34	33	31	33	31	33	31	35	32	34	**33**
MDA_03	30	36	32	34	23	30	26	34	35	36	34	36	31	33	32	**34**
MDA_05	32	34	35	33	27	33	36	32	29	33	31	33	33	35	35	**35**
DDWFSB_10	35	33	36	36	36	32	28	36	36	34	36	34	36	36	36	**36**

Appendix A: Comparison between different failure prediction frameworks

Author	Models	Criteria (Measure)	Result
Panel I: Comparison between traditional statistical models			
Press and Wilson (1976)	LA and MDA models	Correctness of categorical prediction (T1 and T2 errors)	Two models unlikely will give significantly different results.
Collins and Green (1982)	LPM, MDA and LA models	Correctness of categorical prediction (OCC, T1 and T2)	The models produce identical, uniformly results.
Lo (1986)	MDA and LA models	Power of models	There is not differences between models.
Theodossiou (1991)	LPM, LA, and PA models	Correctness of categorical prediction (T1 and T2 errors), Calibration (BS), Information content (pseudo-R^2)	logit model outperforms others; CONFLICT in ranking of others with respect to different measures
Lennox (1999)	LA, PA, and MDA models	Correctness of categorical prediction (T1 and T2)	A well-specified non-linear PA and LA are superior over DA
Bandyopadhyay (2006)	MDA models and logit models	Correctness of categorical prediction (OCC, T1 and T2) Discriminatory power (ROC), Information content (pseudo-R^2, LL)	CONFLICT in rankings using different criteria and measures
Tinoco and Wilson (2013)	logit models taking to accounting different categories of features	Discriminatory power (ROC, Gini, KS), Calibration accuracy (HL)	CONFLICT in rankings using different criteria and measures
Panel II: Comparison between traditional statistical models and survival analysis models			
Luoma and Laitinen (1991)	Cox-hazard, MDA and LA models	Correctness of categorical prediction (T1 and T2)	SA model is inferior than MDA and LA models
Shumway (2001)	Discrete-time SA model, MDA, LA and PA	Correctness of categorical prediction (OCC)	SA model which, encompasses both accounting and market information (respectively, only accounting information) outperforms (respectively, underperforms) other statistical techniques

Panel III: Comparison between statistical models and contingent claims models			
Hilligeist et al. (2004)	BSM-based, LA and MDA models	Information content (LL and Pseudo-R^2)	BSM-based model outperforms both original and refitting version of LA and MDA models
Reisz and Perlich (2007)	BSM-based, KMV, DOC and MDA models	Discriminatory power (AUROC)	DOC and MDA outperforms others for 3-, 5- and 10-year ahead; MDA outperforms others for 1-year ahead failure prediction
Agarwal and Taffler (2008)	Contingent claims based models [HKCL (2004) and BHSH (2008)] and MDA model of Taffler (1984)	Discriminatory power (ROC), Information content (pseudo-R^2, LL), Correctness of categorical prediction (EV for different cost of misclassification)	MDA model outperforms HKCL (2004) on ROC and pseudo-R^2. CONVERSELY, HKCL (2004) outperforms BHSH (2008) and MDA model on LL.
Panel IV: Comparison between contingent claims models and survival analysis models			
Campbell et al. (2008)	A new duration dependent SA without time-variant baseline, SA model [Shumway (2001)] and KMV model	Information content (pseudo-R^2, LL)	The suggested new SA model outperforms both Shumway (2001) and KMV models.
Panel V: Comparison between contingent claims, survival analysis and traditional statistical models			
Wu et al. (2010)	MDA [Altman (1968)], Logit model [Ohlson (1980)], probit model [Zmijewski (1984)] hazard model [Shumway (2001)] and BSM- model [HKCL (2004)]	Information content (pseudo-R^2, LL) Correctness of categorical prediction (OCC), Discriminatory power (ROC)	Shumway outperforms others with respect to LL and Pseudo-R2. Logit model performs better that others with respect to OCC. CONFLICT in rankings
Bauer and Agarwal (2014)	Traditional model, contingent claims based model and hazard model	Discriminatory power (ROC), Information content (LL, R^2) and Correctness of categorical prediction (OCC, T1, T2)	Hazard model outperforms others; CONFLICT in ranking of others with respect to different measures

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